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AN INVESTIGATION OF THE AIR FORCE RISK MODEL

THESIS

THOMAS R. O'HARA, Captain, USAF

AFIT/GCA/LSQ/91S-9

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Presented to the Faculty of the School of Systems and Logistics

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Master of Science in Cost Analysis

THOMAS R. O'HARA, B.S.

Captain, USAF

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Preface

The purpose of this research is to develop a validation/verification methodology for dependent work breakdown structure (WBS) cost element cost risk models. Two general failure modes exist for dependent cost risk methodologies. The first failure mode is when the model fails due to improperly specified input parameters. The second failure mode is when the model fails because the methodology does not properly act on the inputs with valid user specifications.

A specific investigation into the Air Force Risk Model was accomplished. A Comparison Model was developed to determine if and where the model failed. If the model fails then a determination of whether it failed because of the methodology or the implementation must be made. The cost risk methodology affect on twenty-five pairs of triangular distributions is evaluated.

In doing my research, I am greatly indebted to my thesis committee, Capt W. P. Simpson (Ph.D.), Dr. R. Murphy, and Dr. R. Fenno. I am indebted to Mr. J. P. (Pete) Barnum at Los Angeles AFB who suggested the thesis topic. I also thank Capt Fenimore for his WordPerfect® help. Finally, I wish to thank my wife, Cindy, and newborn son, Tommy, for their support, patience and understanding.

Thomas R. O'Hara

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Abstract

This study develops a dependent component cost risk model validation methodology and applies it to the Air Force Risk Model. The validation process consists of ensuring that logically consistent input parameters are acted on properly by the cost risk methodology. Users of all dependent component risk models must be concerned with logically consistent input parameters. Two criteria define logical consistency. The first is the correlation matrix consistency and the second is the consistency between pairs of cost distributions. Three validation criteria are defined and used to validate a cost risk model. The first criterion is that the process must maintain the user defined correlations. The second criterion is that the total cost distribution mean and variance be congruous with the analytical value. The third criterion is that properly specified input parameters not be altered by the cost risk process. A Comparison Model was developed in Quattro® Pro to validate the general Air Force Risk Model methodology. Twenty-five pairs of work breakdown structure cost elements are defined and tested in the Comparison Model. The final research product is a table illustrating the narrow conditions where the Air Force Risk Model is valid.

AN INVESTIGATION OF THE AIR FORCE RISK MODEL

I. *Introduction*

Background

All Department of Defense services are concerned with weapon systems cost. Decision makers want to receive accurate cost estimates, and the cost analyst's goal is to provide accurate information to the decision makers. Furthermore, DoDI 5000.2 (*Defense Acquisition Management Policies and Procedures*) requires that Cost Analysis Improvement Group (CAIG) briefings characterize the cost risk associated with cost estimates (7:13-C-3). Therefore, cost analysts require a tool to evaluate the inherent risk or uncertainty in any weapon system acquisition program. One argument for using a cost risk model based on statistical analysis is that it provides the user with a quantitative justification for resources added (subtracted) from a point estimate as opposed to a simple factor applied to all estimates. This research develops a cost risk validation/verification methodology and applies it to a probabilistic/statistical cost risk model.

Total cost estimates are the summation of the lower level work breakdown structure cost elements. A Work Breakdown Structure (WBS) is defined by Military Standard

881A (MIL-STD-881A) as

> a product-oriented family tree composed of hardware, services and data which result from project engineering efforts during the development and production of a defense materiel item, and which completely defines the project/program. A WBS displays and defines the product(s) to be developed or produced and relates the elements of work to be accomplished to each other and to the end product. (17:2)

The WBS is broken down into levels. Cost estimates are usually developed at the level 3 or lower. The following definitions of WBS levels are from MIL-STD-881A:

> Level 1 is the entire defense materiel item: for example, the Minuteman ICBM System, the LHA Ship System, or the M-109A1 Self-Propelled Howitzer System. Level 1 is usually directly identified in the DoD programming/budget system either as an integral program element or as a project within an aggregated program element.
>
> Level 2 elements are major elements of the defense materiel item: for example, a ship, an air vehicle, a tracked vehicle, or aggregations of services, (e.g., systems test and evaluation); and data.
>
> Level 3 elements are elements subordinate to level 2 major elements: for example, an electric plant, an airframe, the power package/drive train, or type of service, (e.g., development test and evaluation); or item of data (e.g., technical publications). (17:2-3)

An example from MIL-STD-881A of the WBS levels is shown in Figure 1. The air vehicle, training, and peculiar support equipment are the first three Level 2 breakouts. These are further subdivided into their respective Level 3 breakouts as shown. A similar work breakdown structure is available in MIL-STD-881A for other Air Force weapon system types as well as Army and Navy weapon systems. This breakout provides logical order to cost estimating and also

Level 1	Level 2	Level 3
Aircraft system		
	Air vehicle	
		Airframe
		Propulsion unit
		Other propulsion
		Communications
		Navigation/guidance
		Fire control
		Penetration aids
		Reconnaissance equipment
		Automatic flight control
		Central integrated checkout
		Antisubmarine warfare
		Auxiliary electronics equipment
		Armament
		Weapons delivery equipment
		Auxiliary armament/weapons delivery equipment
	Training	
		Equipment
		Services
		Facilities
	Peculiar support equipment	
		Organizational/intermediate (Including equipment common to depot)
		Depot (Only)

Figure 1 MIL-STD-881A Work Breakdown Structure (First 3 Level 2 breakouts) (16:17-18)

performs the function of maintaining some consistency in cost estimating structure between various programs. Cost estimates may actually be developed at lower than the 3rd level, which provides additional detail into the cost estimate.

The total cost point estimate is the summation of all lower level point estimates. The point estimate is usually

interpreted as the mean for each cost element. Typically the cost estimating process is the summation of cost element means to generate the total cost point estimate.

The *Air Force Systems Command Cost Estimating Handbook* defines cost risk as follows:

> Risk and uncertainty refer to the fact that, because a cost estimate is a prediction of the future, there is a chance that estimated cost may differ from actual cost. Moreover, the lack of knowledge about the future is only one possible reason for such a difference. Another equally important cause is the error resulting from historical data inconsistencies, cost estimating equations, and factors that are typically used in an estimate. (2:13-1 to 13-2)

Cost risk analysis is the quantification of estimating methodology uncertainty in the total cost distribution. There is some uncertainty with any estimate. From Jago, the analyst has many tools available to generate component cost estimates. Cost risk analysis is a tool available to account for some of this uncertainty (12:4). Since cost risk analysis is another prediction, it only quantifies the confidence in the estimate.

Cost risk analysis is applied to cost estimates through the WBS. There is a distribution of cost for each WBS cost element. Each cost element has an associated probability density function (p.d.f.). The probability density function represents the distribution of probability for an event occurrence (20:187). The point estimate or mean cost for a WBS cost element will vary as a function of the methodology used in generating that cost estimate.

According to Murphy, typically the cost analyst will use the mean cost estimate based on the method that is most applicable to the subsystem and the weapons program. In applying the cost risk methodology, the WBS element mean cost is interpreted as the most likely cost estimate. The lowest likely and highest likely cost are determined by the prediction interval around the mean cost. The prediction interval level is left for the user to decide. That is, should the prediction interval capture 80%, 90% or 99% of the cost estimate with that particular cost estimating methodology (18)? Neter, Wasserman and Kutner define the prediction interval as the area under the prediction probability density function for a given mean. For example, a cost estimate $\pm 3\sigma$ would be a 99.87% prediction interval around the mean ($N(\mu, \sigma^2)$). The highest (lowest) likely cost estimate would be at the $+ 3\sigma$ ($- 3\sigma$) point (19:80-81).

Cost estimating risk analysis is the function that cost analysts perform before they present the point estimate to decision authorities. The total cost point estimate from cost risk analysis represents the median cost for a weapon system. The cost risk process uses the mean cost of lower level elements to determine the median total cost. Typically analysts will report two costs along a cumulative probability distribution function (c.d.f.) at the fifty and seventy percent probability levels. The cumulative probability distribution function expresses the probability that a cost does not exceed a specified value (20:185).

From the *AFSC Cost Estimating Handbook*, the fifty percent confidence level from the cost risk process represents the median value of the total cost distribution, which means there is a fifty percent probability that actual cost will exceed the estimated cost (2:13-13). Similarly, the seventy percent confidence means that there is a thirty percent probability of exceeding the cost estimate. Cost risk analysis techniques assume that the program remains constant as it quantifies uncertainty in the cost estimating methodology (2:A-16, 13-1 to 13-2). It does not account for Congressional actions, strikes, or natural phenomena that occur unexpectedly.

Cost risk for the total system is defined by using the p.d.f./c.d.f. for total system cost. The method of generating the total system cost p.d.f./c.d.f. depends on the assumptions of cost dependency and the shape of the WBS cost element distributions. The amount of estimate confidence is indicated by the total cost distribution cumulative distribution function.

Cost risk analysis methodologies (refer to Figure 2) rely on the definition of cost distributions for each cost element. The analyst needs to define the mean, lowest likely cost, highest likely cost, variance, distribution shape and pairwise correlations (correlation coefficient, ρ) (12:1-12).

The mean and variance of total cost can be determined analytically (19:5-6). Cost risk methodologies must either

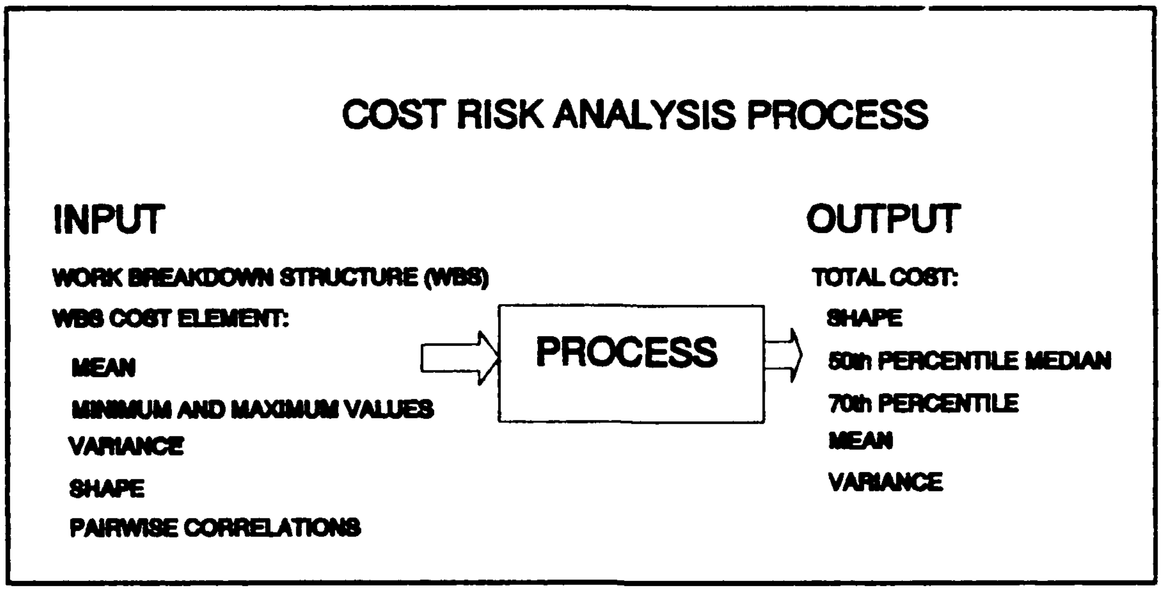


Figure 2 The cost risk analysis process

make assumptions about the shape of the total cost distribution or determine the shape by simulation methods. Therefore the cost risk analysis process is the summation of all lower level cost distributions. The summation of independent probability density functions to determine the total probability density function shape can be accomplished with convolution (21:317).

Convolution of probability density functions may be calculated with at least two methods: analytically and simulation. The reader interested in analytical methods should reference any general statistical/probability text such as Parzen's *Modern Probability Theory and Its Applications* (21:317). The simulation convolution method sums one sample from each distribution to form the sum of the total cost distribution for several samples (100 to 1000 samples) (2:13-29 to 13-32).

The most commonly used cost risk methodology within the Air Force is the Air Force Systems Command (AFSC) Risk Model. The AFSC Risk model assumes that all cost elements within the WBS are statistically independent. By assuming that each subsystem element is independent, the model misestimates the total program cost if the cost elements are dependent. Garvey states that if all weapon subsystem correlations are positive, then the total cost is underestimated (10:5).

Unfortunately, WBS cost element dependencies exist. From Murphy, weapon system component costs are driven by the physical and performance parameters that describe the system. The physical and performance parameters are driven by the threat for which the weapon system is designed. Therefore, the overall system characteristics are relatively constant to the threat. However, intrasystem trades do exist while maintaining the same overall goal. The physical characteristics for each component will have a specific interrelationship for any given weapon system. These interrelationships drive the cost correlations. Therefore, the cost correlations are not spurious statistical relationships (18).

Weapon subsystem cost dependencies can be further understood with two simple examples. When estimating an aircraft the WBS may include the level 3 elements airframe and propulsion unit. If the weight of the airframe is increased (thus increasing the cost) then the propulsion

unit must be increased in some way to handle this increased weight. In very general terms, the propulsion system power is increased to meet the increased demand of weight. Again in very general terms, both actions would most likely increase the cost of their respective subsystems. This would indicate a positive cost correlation between these cost elements.

Cost correlation relationships are not always positive. Some subsystems cost's decrease as another subsystem increases in cost. Consider a target seeking missile such as a kinetic kill vehicle (Strategic Defense Initiative) or an air-to-air missile. The WBS for this weapon system would include some type of sensor (active or passive) and a propulsion system. If the sensor acquires the target at a greater range, the propulsion system does not have to produce as much energy as a sensor that detects a target at a shorter range. The sensor that detects the target at greater range is more expensive than the sensor that detects its target at short range. Also the propulsion system that produces greater energy is more expensive than one with less energy. There is a negative cost correlation exhibited by this example. As one subsystem increases in cost the other subsystem decreases in cost.

Devaney and Popovich showed in their research that the cost dependency between weapon system components should be an important consideration in cost risk models. Cost risk

analysis techniques have traditionally assumed that the WBS elements are statistically independent (8:77).

There are several methods available to evaluate risk. Garvey and Abramson & Young developed analytical cost risk models. Garvey's model is called the Analytic Cost Probability (ACOP) model and Abramson's and Young's model is called the Formal Risk Evaluation Methodology (FRISKEM) (1:1: 10:1). Both works will be reviewed in Chapter II. This thesis will concentrate on the Air Force Risk Model (referred to as the Tecolote Risk Model in this study) which is a new model under development by Tecolote Research Inc. and contracted by the US Air Force Cost Center (AFCC). The Air Force Risk Model is designed to estimate cost risk in the presence of cost dependencies or correlations between WBS weapon subsystems (12:9-10).

Verification and Validation

Verification and validation are defined by Law and Kelton as:

> Verification is determining whether a simulation model performs as intended, i.e., debugging the computer program...Validation is determining whether a simulation model (as opposed to the computer program) is an accurate representation of the real-world system under study. (14:333-334)

Banks and Carson define verification and validation as:

> Verification pertains to the computer program prepared for the simulation model. Is the computer program performing properly? Validation is the determination that a model is an accurate representation of the real system. (3:14)

This research will determine under what conditions a risk model methodology is valid. Furthermore, by comparing the output of any model with another user defined model it will verify the methodology's implementation.

Validation Criteria

The validation process is exhibited in Figure 3. There are two types of risk model failure modes. The first failure mode (further divided into failure modes 1a and 1b) occurs when inputs are not properly specified. This is the user's burden. That is, the user is responsible for specifying proper inputs. The second failure mode (failure mode 2) occurs when the methodology does not properly act on correctly specified user inputs. This is the failure due to the model's methodology. The first failure mode is subdivided into two types of failures. The first subdivision is failure mode 1a and it is when the correlation matrix is not internally consistent. The second subdivision is failure mode 1b and it is when the cost element distributions are not consistent with the user specified correlation matrix. Once the input parameters fail at 1b, the user may change either the shape of the distribution or the pairwise correlation. The remainder of this research assumes that the shapes are changed to the correlation. However, changing the correlation to the shape is equally valid. A set of criteria (described later in this chapter) can be developed to validate the model in

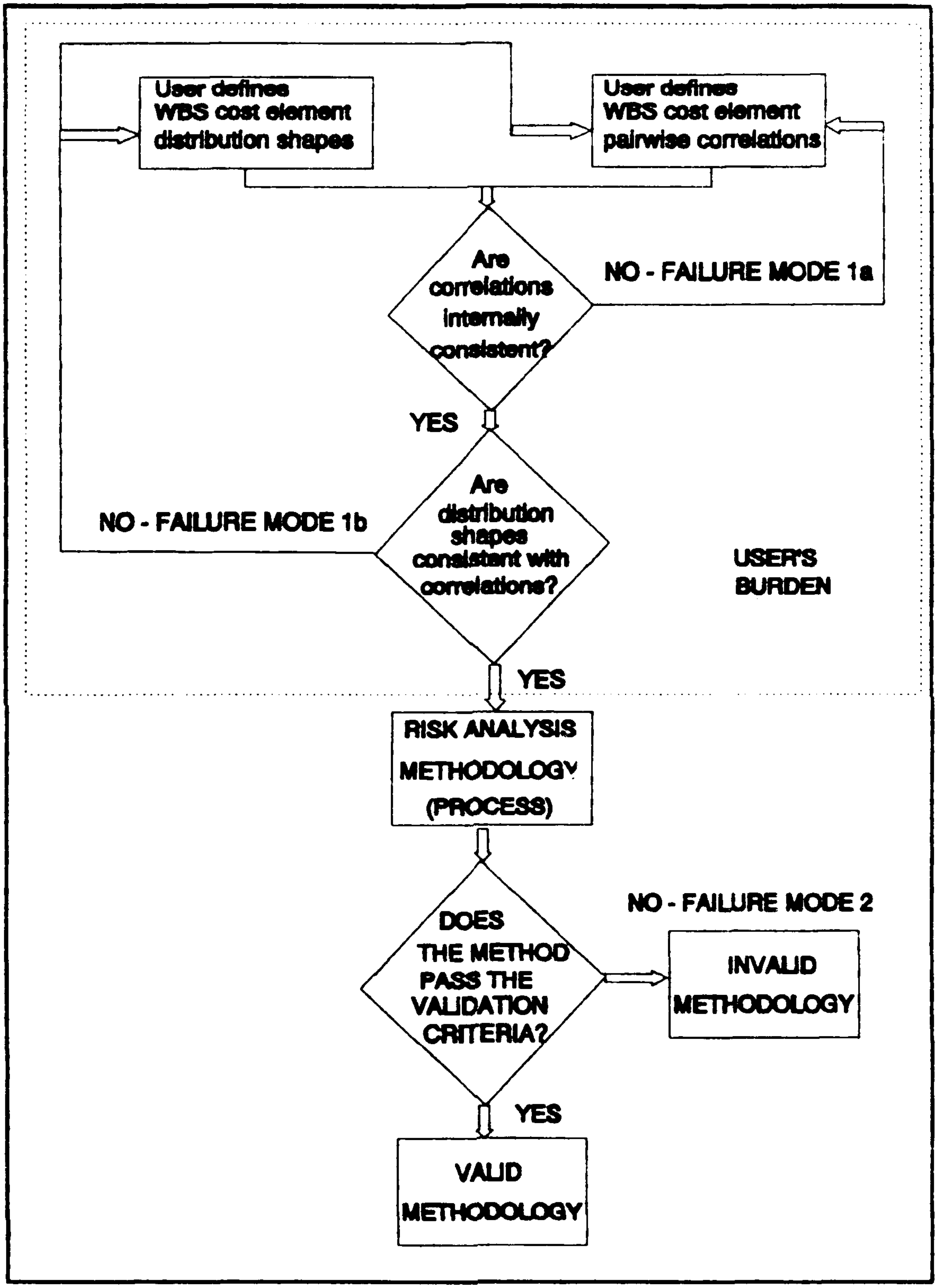


Figure 3 Cost risk methodology validation criteria

reference to failure mode 2. The user should be advised when he/she misspecifies parameters or when the model fails to act properly on the user's specifications.

Failure mode 1a is understood with a simple correlation matrix example. Murphy describes a three element WBS, elements A, B, and C. Element A has a high positive correlation with both elements B and C. This relationship forces a positive correlation between elements B and C. The correlation matrix would then be logically consistent (18).

The question of consistency within the correlation matrix is fairly easy to verify. Searle states that the correlation matrix is non-negative definite (either positive semi-definite or positive definite) (23:348-349). The test for positive definiteness and positive semi-definetness is covered in chapter III.

Failure mode 1b requires highly correlated WBS cost element distributions to have approximately the same shape. This is evident with three simple examples. First consider Figure 4, the case of two identically distributed cost element distributions. If these distributions are correlated, the correlation should be positive. Any negative correlation would be inconsistent with the cost distribution's shape (18). As the cost of one element increases, the other element should increase or remain constant (19:5, 522). Consider Figure 5 with two WBS cost element distributions, both right triangles, one skewed right and the other skewed left. Thus, these are opposing

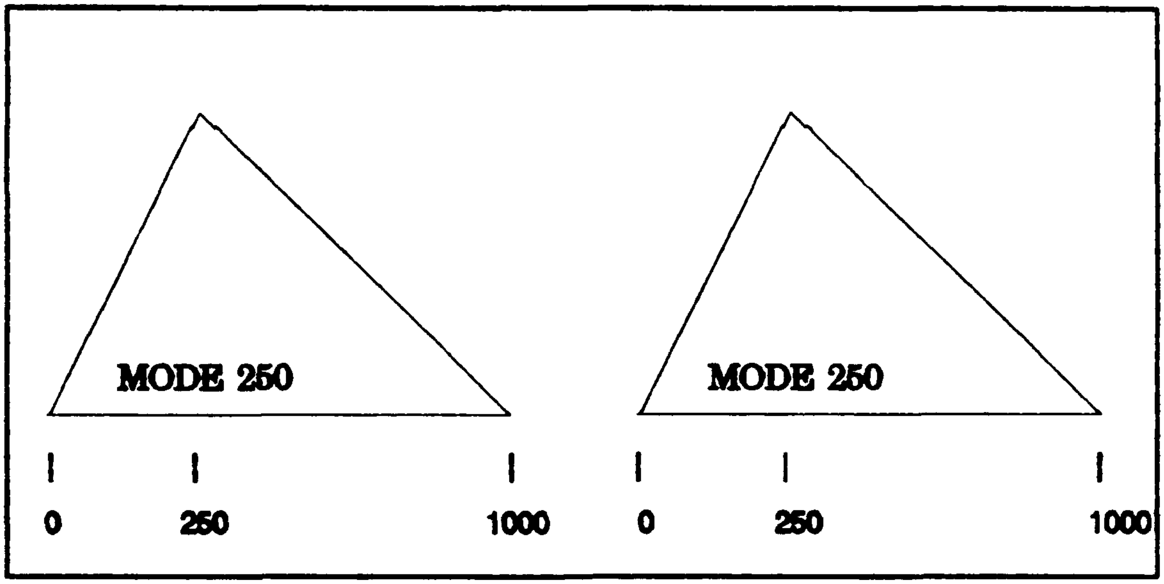


Figure 4 Two identical triangular cost distributions

right triangular distributions. Any positive correlation would be an inconsistent user specification (18). As the cost of one element increases, the cost of the other should decrease or remain constant (constant would assume that the two cost elements are statistically independent). This is an assertion from basic statistical theory relating correlation and covariance (19:5, 522). A third possibility is of symmetrical cost distributions (Figure 6). Two symmetrical distributions may be either positively or negatively correlated. If the two distributions are positively correlated, then the costs change in the same direction. However, an equally valid possibility is that one will decrease in cost as the other increases in cost (negative correlation). Since an equal area under the cost element probability density function is covered during the change, the correlation consistency remains valid (18). The

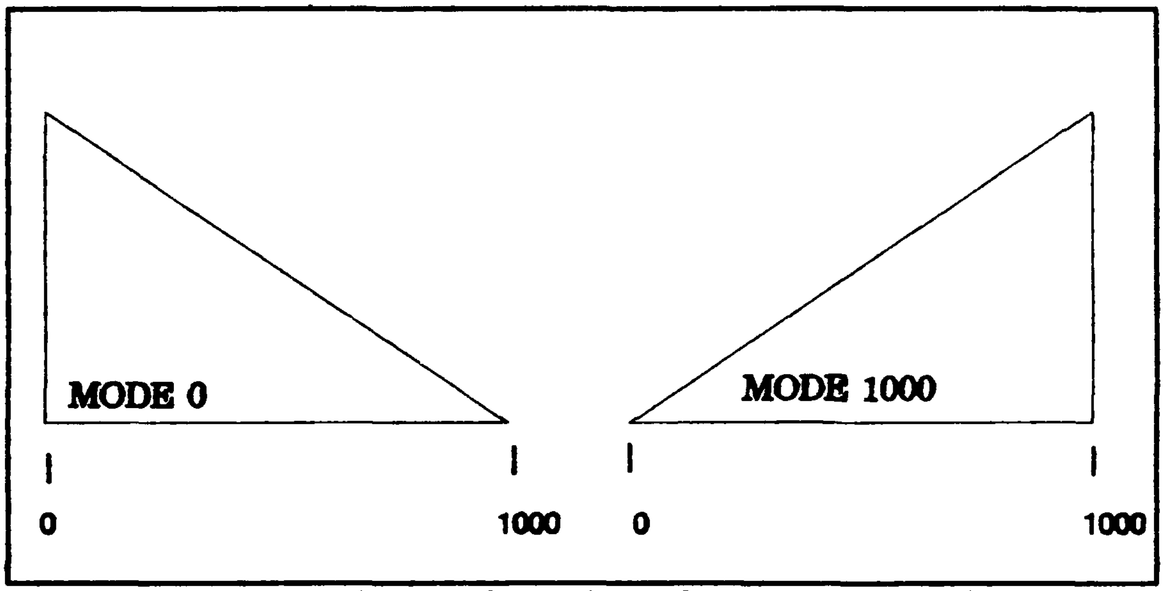


Figure 5 Two opposing right triangular cost distributions

user should not misunderstand that cost distributions must have some correlation. Cost element distributions may also be independent. Also the user needs to be aware that there is a large "gray" area where there is not such a clear cut difference between logical and illogical correlated distributions. The three cases stated above are simple examples for illustration purposes. Distributions found in the real world will be much more complex, and the analyst should take great care in applying any risk methodology that considers dependency among components.

There are three criteria that identify Failure mode 2. A valid cost risk methodology will pass all criteria. The criteria that identify failure modes are: 2a. The user defined component correlations should be maintained through the cost risk model (i.e., input ρ = output ρ). 2b. The total cost mean and variance calculated by the cost risk

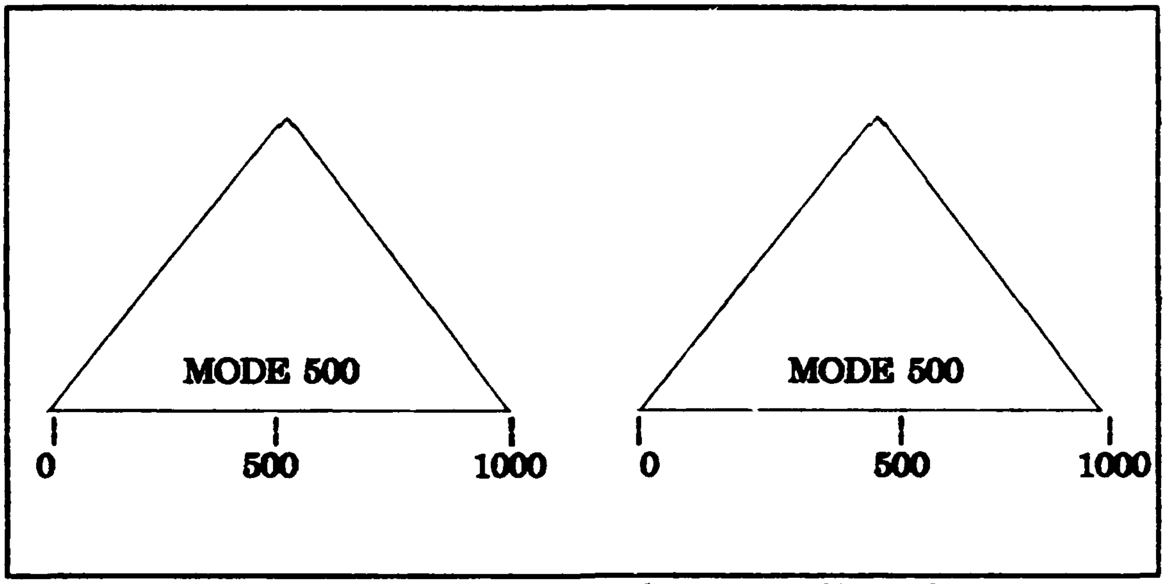


Figure 6 Two symmetrical triangular cost distributions

model should be equal to the analytical total cost mean and variance. 2c. The input WBS cost element probability density function shapes should be the same as the output shapes.

The user defined correlations must be applied to the cost distributions through the cost risk analysis process. The validation of this is accomplished by criterion 2a. The Tecolote Risk Model methodology satisfies validation criterion 2a as shown by Book and Young in their paper at the 24th Annual DoD Cost Symposium (4:11). The results of their research will be shown in Chapter IV.

The total cost mean and variance may be derived analytically (19:5-6). The cost risk process should calculate the same values as calculated analytically. Criterion 2b may be confirmed by simply comparing the

summary statistics of the simulation output values with the analytically determined values.

The total cost distribution should behave in the manner as stated by standard statistical methods. Basic statistical theory as discussed by Neter, Wasserman and Kutner show the statistical relationships between random variables used in risk an..lysis. The cost element distributions may be summarized by two statistics. The first is the mean of the sums is the sum of the means.

$$E\{\sum_{i=1}^{n} Y_i\} = \sum_{i=1}^{n} E\{Y_i\} \tag{1}$$

and the second is the variance of the sums is equal to the sum of the variances plus two times the covariances.

$$\sigma^2\{\sum_{i=1}^{n} Y_i\} = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma\{Y_i, Y_j\} \tag{2}$$

Specifically, for n = 2 the relationships are:

$$E\{Y_1+Y_2\} = E\{Y_1\}+E\{Y_2\} \tag{3}$$

and

$$\sigma^2\{Y_1+Y_2\} = \sigma^2\{Y_1\} + \sigma^2\{Y_2\} + 2\sigma\{Y_1, Y_2\} \tag{4}$$

The dependency between pairs of random variables is indicated by the covariance, represented by $\sigma(Y_1,Y_2)$, and is defined as follows:

$$\sigma\{Y_1, Y_2\} = E\{(Y_1 - E\{Y_1\})(Y_2 - E\{Y_2\})\} = E\{Y_1 Y_2\} - (E\{Y_1\})(E\{Y_2\}) \tag{5}$$

Correlation, represented by ρ, is the standardized covariance between two WBS cost elements. This is represented by the following equation:

$$\rho\{Y_1, Y_2\} = \frac{\sigma\{Y_1 Y_2\}}{\sigma\{Y_1\}\,\sigma\{Y_2\}} \tag{6}$$

The equations shown are from Neter, Wasserman and Kutner (19:5-6, 522).

Criterion 2c may be confirmed by comparing the input and output distributions of the cost risk process with a goodness of fit test. The analyst using any risk model should expect to get the same distribution out of the risk analysis process that the analyst inputs. According to Murphy, the cost distributions that are used as inputs already have history. That is, they are already correlated to each other. It is difficult to develop cost distributions independently of each other (18). Thus if the user inputs a triangular distribution, he/she should in return be generating random deviates (variates) from a triangular distribution. This research will test pairs of distributions with a range of correlations. It will show at what correlation the cost risk methodology fails to produce similar post cost risk analysis process distributions.

The above criteria may be used to validate any risk methodology which considers dependencies between WBS cost

elements. This is exhibited in Figure 3 by showing that the user first defines the parameters and then the user verifies that they are consistent in both pairwise correlations and distribution shapes.

A Comparison Model is developed to accomplish criterion 2c. This model uses the same methodology as described by Jago (12:1-12) and Book & Young (4:1-19). The Tecolote Risk Model was not available for this research and the executable code did not offer the necessary research data.

Verification Criteria

The Tecolote Risk Model will be verified using validation criterion 2b. The total cost mean and variance will be compared to the analytical values. The Air Force Risk Model computer program was not available for this research; therefore, criteria 2a and 2c could not be accomplished.

Specific Problem

The Tecolote Risk Model is a Monte Carlo model that uses Cholesky decomposition to transform independent random deviates (variates) to dependent random deviates according to the user specified correlations. The focus of this research is to apply the validation methodology to the Tecolote Risk Model. Furthermore this research will investigate the validity of the Cholesky decomposition as a risk analysis WBS cost element correlation methodology. The

specific task of this research is to apply the validation criteria (Failure Mode 2) to the output generated from valid user specifications (inputs that pass Failure Mode 1).

Hypothesis

The hypothesis will test the Tecolote Risk Model for the three criteria using logically consistent correlations and distributions. The hypothesis test is:

H_0: The Tecolote Risk Model is a valid Methodology

H_a: The Tecolote Risk Model is not a valid Methodology

II. *Literature Review*

Overview

With one exception, this review of the literature will summarize cost risk analysis techniques that consider dependency among cost elements. The exception is the general Monte Carlo Method model. These techniques may be divided into two general categories: analytical and simulation methodologies. This chapter will also cover the definitions of cost risk analysis and cost contingency analysis.

Analytical risk analysis techniques available now are those that make assumptions about the shape of the total cost distribution and use standard statistical formulas to provide the cumulative probability on the c.d.f. The Analytical Cost Probability (ACOP) Model assumes that the total cost distribution is a normally distributed variable (10:5). The Formal Risk Evaluation Methodology (FRISKEM) Model assumes that the total cost distribution is a lognormally distributed variable (1:4).

Simulation methods generally use the Monte Carlo method to derive the total cost distribution by sampling the input distributions and then use convolution to obtain the shape of the total cost distribution. Convolution is a mathematical method of summing two or more statistically independent probability density functions. The Air Force Systems Command Risk Model uses this technique for

independent WBS cost distributions. The Tecolote Risk Model uses convolution in addition to Cholesky decomposition to correlate the independent random deviates (variates) in the Monte Carlo simulation to form the total cost distribution (12:9-12). The Tecolote Risk Model is thus a series of steps, which are generate statistically independent random deviates, standardize random deviates, correlate independent random deviates using the Cholesky decomposition, destandardize random deviates, and then sum the lower level cost elements using convolution to form the total cost distribution (12:4-12).

Devaney and Popovich, in their literature review in 1985 showed that existing models either ignored cost dependencies or assumed that there was total cost dependence. In either case, the total cost is misestimated. However, by doing cost risk analysis under both assumptions, independence and total positive dependence, the risk analysis output will typically provide a bound to the true estimate (8:14-29).

General Definitions. Jago defines the four elements of uncertainty that the AF Risk model considers. The elements are estimating, scheduling, technology, and configuration uncertainties and are defined as:

> **Estimating uncertainty** establishes a band around an estimate showing the probable error in the estimate. It is measured in units of cost. Estimates for the elements of a Work Breakdown Structure are developed by a variety of methods, each with its own characteristic estimating uncertainties.
>
> Four basic estimating methods are now in common use. These are: (1) Cost Estimating Relationships

(CERs), (2) Factors, (3) Analogies, and (4) Engineering Build-Up. CERs and Factors can be grouped since they are statistical in nature and represent expected values derived from some data base. Similarly, Analogies and Engineering Build-Up are based on discrete data points.

Schedule uncertainty specifies a band of time usually as durations or dates. The types of information an analyst has for the schedule estimation problem are things like when the program starts, intermediate milestone dates (such as PDR or CDR), and projected program completion. Program schedules usually come in the form of Gantt charts or networks. Schedule uncertainty translates to cost uncertainties when activities are on the program's critical path, for schedules containing a high level of concurrence or parallel paths, and for labor intensive activities (such as programming).

Technology uncertainty cannot be measured directly in either cost or time, but rather in terms of the number of remaining unresolved technical issues. A good surrogate would measure its impact on successfully achieving critical program milestones on schedule. Viewed in this light, technology uncertainty impacts schedule uncertainty when the technology is not mature when needed, or when the subsystem design incorporating the technology does not adequately reflect its technical performance or interface characteristics.

Configuration uncertainty captures the changes in basic cost-driving variables. Thus, if the cost-driving variables were weight, volume, or power, then the units of measure might be in pounds, cubic feet, or kilowatts. The sources of configuration uncertainty are design changes during development or production, or growth in the cost-driving variables from 'requirements creep'. (12:4-5)

To limit the extent of this research, cost estimating uncertainty (referred to as cost risk in this research) is the primary focus of this research. Any analyst must be very careful in accounting for the remaining three uncertainties. By ignoring the schedule, technology, and configuration uncertainties, the analyst will not capture

the true risk in an estimate. This author recommends that further research or guidelines be developed before the remaining three categories are implemented.

Contingency

The *AFSC Cost Estimating Handbook* defines contingency as:

> an allowance or amount added to an estimate to cover a possible future event or condition arising from presently known or unknown causes, the cost outcome of which is indeterminable at a present time. (2:A-16)

and contingency analysis as follows:

> Repetition of an analysis with different qualitative assumptions - e.g. how well will equipment perform on different terrain/type of conflict, etc. (2:A-16)

Contingency allowances are different than resources added (subtracted) due to risk analysis techniques. A contingency budget could be used for anticipated budget cuts, congressional cuts, and other unknown problems. Risk budgets are strictly to compensate for known problems with the cost estimating methodologies. Contingency analysis may be said to be a what-if exercise to generate multiple program options to present to a decision maker (2:A-16). This thesis will not cover contingency analysis or the techniques available for doing it.

Statistical and Probabilistic Relationships

WBS element cost distributions may be described by summary statistics such as the mean, mode, and variance.

Another way to describe WBS element cost distributions is graphically or functionally. This second description determines the actual shape of any probability density function (p.d.f.) from which the mode, mean, and variance may be derived. The Normal distribution is the only distribution that is completely defined by the mean and variance. All other distributions require additional moments to fully describe the shape (8:26). WBS element interrelationships are described by the pairwise correlation terms between the elements (19:522).

The summation of the means and variances of lower level cost elements will result in the total cost mean and variance (see equations 1 and 2). However, this does not provide the shape of the total cost distribution. Convolution is a mathematical method that computes the shape of independent distributions analytically (6:85-88).

Analytical Cost Risk Methodologies

Convolution Overview. The shape of the total cost distribution may be found by simulation or analytical methods. Simulation methods are discussed in Chapter III. The analytical convolution method may be used to determine the exact summation of independent distributions; however, it is possible that the solution does not exist (16:68-82). Therefore, simulation (specifically the Monte Carlo Method) methods offer a practical solution to convolution (14:50).

Analytical Cost Probability Model (ACOP). Garvey develops and provides an example of a weapon system acquisition cost risk model. The model is called the Analytic Cost Probability (ACOP) model and was developed at the MITRE Cost Analysis Technical Center (10:1).

Garvey stated that the Air Force Systems Command's Electronic Systems Division requires two properties in a cost risk model. First, the model must be a non-simulation risk model and second, it must take into account the effect of (WBS) element interdependencies. Most cost risk models are based on a Monte Carlo (simulation) method and furthermore assume that all WBS elements are statistically independent (1[illegible]-6).

Garvey showed that a closed form solution would alleviate some of the restrictions in implementing a simulation cost risk model, primarily the long computation time required for typical Monte Carlo methods. This model requires definition of the WBS element's distribution type, most likely cost, standard deviation and WBS element pairwise correlations. The ACOP model assumes that the level 2 prime mission equipment is a normally distributed variable with all other level 2 cost elements correlated to it (10:3-10).

MIL-STD-881A defines prime mission equipment for electronic systems as:

> The prime mission equipment element refers to the equipments and associated computer programs used to accomplish the prime

> mission of the defense materiel item. Those support equipments and services vital to the operation and maintenance of the system, but not, integral with the prime function of the system are excluded. (17:34)

Garvey states that if the prime mission equipment cost dominates the cost of all other level 2 WBS elements, then it can be assumed that total system cost is approximately normal (10:1-11). The assumption of a normally distributed variable is the key limiting factor with the ACOP model. The cost analyst must assume a shape for the total cost distribution.

Garvey provides an appendix with proofs for all theorems used throughout the model. The author also provides an example to illustrate the methodology (10:1-11).

Garvey's model alleviates the necessity of using simulation methods. The ACOP model and the Tecolote Risk model are similar in that they allow for the input of WBS element correlation. The model's ability to include WBS correlations should provide better program cost estimates (10:1-11). However, the cost analyst must understand the limitations of the model discussed in the conclusion of this chapter.

Formal Risk Evaluation Methodology (FRISKEM). Abramson and Young define a model which may be used to evaluate multiple program options including risk analysis. This model is called the Formal Risk Evaluation Methodology (FRISKEM). They also discuss the possibility to generalize the model for standard risk analysis. However, different

assumptions (these assumption are not developed in the paper) about the element distributions must be made (1:1-9).

Abramson and Young define the FRISKEM as a model which assumes that lower level WBS cost element distributions are triangular cost distributions. The sum (total cost distribution) of these lower level elements is assumed to be a log-normal distribution. This model has been developed to compare competing program solutions to the same general problem (1:8-9).

Simulation Cost Risk Methodologies

General Monte Carlo Methods. Law and Kelton define the Monte Carlo simulation method as

> a scheme employing random numbers, that is, U(0,1) random variables, which is used for solving certain stochastic or deterministic problems where the passage of time plays no substantive role. Thus, Monte Carlo simulations are generally static rather than dynamic. (14:49)

Dienemann describes the Monte Carlo technique required for cost uncertainty analysis. The model requires that the user input the summary statistics of the WBS elements. He uses the Monte Carlo method to generate samples from that distribution. The samples are summed by convolution to generate the total cost distribution. These methods are developed and an example of usage is shown. The model assumes that all WBS elements are statistically independent (9:1-27).

Monte Carlo Convolution. From Jago and Book & Young, convolution is used in Tecolote Risk model by summing the

random deviates from each lower level WBS cost distribution to form the total cost distribution (12:12; 4:14). According to Murphy, one random deviate from a cost element distribution represents one sample of cost from that distribution. Thus the total cost distribution shape is formed by the summation of all lower level cost element cost samples 1000 (default value in the Tecolote Risk Model) times (18).

Correlated Monte Carlo Methods. Johnson describes the use of the Cholesky decomposition (factor) for normally distributed variable in Monte Carlo models. The method described generates correlated normal variates from independent normal variates. He states that this method will only work when the correlation matrix is nonsingular (that is it is invertible). Johnson also indicates that the Cholesky factor is not unique. There are other factorizations which solve $\mathbf{AA'} = \Sigma$, where Σ is the correlation matrix (13:52-55).

Tecolote Risk Model Overview. The Tecolote Risk Model has been designed to consider four types of uncertainties. These are estimating (cost risk), schedule, technology, and configuration uncertainties (12:4-5). This research will concentrate on cost estimating risk exclusively.

The Tecolote Risk Model requires the following inputs for each WBS element or major subsystem: most likely cost, highest likely cost, lowest likely cost, distribution type (beta, triangular, and uniform), standard deviation, and

subsystem pairwise correlations (12:3-12). The Tecolote Risk Model is a Monte Carlo model that uses Cholesky decomposition to transform independent random deviates to dependent random deviates according to the user specified correlations. The Monte Carlo random deviates are shaped by the user defined input parameters into the Tecolote Risk Model. The Cholesky decomposition is applied to the user defined WBS cost element correlation matrix. The Cholesky decomposition will be further discussed in Chapter III. The Cholesky decomposition is a numerical method that factors a symmetric positive definite matrix into upper and lower triangular matrices (11:141-146). Positive definiteness will be discussed in Chapter III. The Tecolote Risk Model uses convolution to generate the total program cost distribution (12:11-12).

The Cholesky factor (an n x n matrix) is postmultiplied by the independent Monte Carlo random deviates. This forms correlated Monte Carlo random deviates (12:11). The first distribution is never changed, but all subsequent cost element distribution shapes are changed dependent on the pairwise correlation defined by the user. If the correlation matrix is the identity matrix (meaning that the distributions are independent), then the post factored distributions are identical to the pre-factored distributions.

The correlated Monte Carlo draws are summed to form the total cost distribution (12:12). This final distribution

forms a p.d.f. and c.d.f. From the total cost distribution, the decision maker may select the confidence level and thus the cost estimate he/she wishes to report.

Conclusions

The problem with using the analytical risk analysis techniques (ACOP and FRISKEM) discussed above is that both models assume a specific shape for the total cost distribution. This limits the applicability of the model to a subset of all possible total cost distributions. The Tecolote Risk Model does not assume a shape about the total cost distribution.

The ACOP model may be used for situations where the prime mission equipment is normally distributed and dominates all other cost elements correlated to it. ACOP will not be further discussed in this research.

The Formal Risk Evaluation Methodology (FRISKEM) is a potential methodology for comparisons of multiple program solutions. However, an evaluation of the assumptions of distribution types would be required for departure from those rigid guidelines. FRISKEM will not be further discussed in this research.

The use of a simulation model appears to be the most appropriate approach to cost risk analysis. The major concern about the Tecolote Risk Model is that it uses valid correlation methodologies. The Cholesky decomposition will be more fully explored and developed in chapter III.

III. *Methodology*

Overview

This chapter will provide an overview of the Tecolote Risk Methodology and how the methodology is implemented in the Comparison Model. The risk methodology/model validation and verification methodology will be described in this chapter.

The methodology developed in this research is general and may be applied to all cost risk models which consider cost element dependencies. However, the methodology is applied in this research specifically to the Tecolote Risk Model.

The Tecolote Risk Model

The Tecolote Risk Model generates uniform random deviates, forms these into user defined p.d.f.s (beta, triangular, and uniform), standardizes (normalizes) the random deviates, computes the Cholesky lower triangle factor, postmultiplies the Cholesky factor by the standardized random deviates, and then destandardizes them to form the post factored distributions. The total cost of all lower level cost elements is calculated by convolution. In simulation models convolution is simply the addition of the vectors of random deviates (14:249-250). According to the *Air Force Systems Command Handbook*, each random deviate is a cost sample (also referred to as draw) from its

respective distribution. Therefore, if the model has two cost elements, then one sample from the first distribution of cost is added to the corresponding sample from the second distribution of cost. The summation of one sample from each WBS cost element distribution is a sample of cost from the total cost distribution. The collection of these samples forms the total cost distribution (2:13-29 to 13-32). The Tecolote Risk Model default number of random deviates is 1000 with a maximum number of random deviates of 9,999 (12:45).

Correlation Matrix

Positive Definite Matrices. Stoer and Bulirsch define positive definite matrices as follows:

> A n x n matrix C is said to be positive definite if it satisfies:
> (a) $C = C^H$, i.e., C is a Hermitian matrix.
> (b) $x^H Cx > 0$ for all $x \in \mathbb{C}^n$, $x \neq 0$. (24:172-173)

> A Hermitian matrix C is positive definite (positive semidefinite) if and only if all eigenvalues of C are positive (nonnegative). (24:330)

Searle states that the correlation matrix is non-negative definite (either positive semi-definite or positive definite). This is due to the fact that all variances in the variance-covariance matrix are 0 or positive (23:347-349). Searle states that "symmetric matrices are a subset of Hermitian matrices" (23:342). Since the correlation matrix (C) is symmetric, it is also known to be Hermitian. Thus, testing for positive definiteness (positive

semidefinite) is simply a computation of the correlation matrix eigenvalues. According to Stoer and Bulirsch, if all the eigenvalues are strictly positive (non-negative), then the matrix is positive definite (positive semidefinite) (24:330). Thus the test for valid correlation matrices is simply a calculation of the correlation matrix eigenvalues. If all eigenvalues are non-negative, then the correlation matrix is valid.

Cholesky Decomposition. The correlation matrix, C, is defined to be a real symmetric positive definite matrix. Then, Stoer and Bulirsch describe the Cholesky decomposition (also referred to as Cholesky factorization in some texts) as the operation that results in finding L, the lower triangle factor matrix of a symmetrical positive definite matrix, C. Then $C=LL^T$ where L is the lower triangle factor and L^T is the upper triangle factor. Formally, Stoer and Bulirsch define Cholesky decomposition as follows:

> For each n x n positive definite matrix C there is a unique n x n lower triangular matrix L (l_{ik} = 0 for k > i) with l_{ii} > 0, i = 1,2,...,n, satisfying $C = LL^T$. If C is real, so is L. (24:174)

Specifically, if C is defined to be a real 3 x 3 correlation matrix, then the Cholesky decomposition is:

$$C=\begin{bmatrix} 1 & \rho_{21} & \rho_{31} \\ \rho_{21} & 1 & \rho_{32} \\ \rho_{31} & \rho_{32} & 1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix} \tag{7}$$

Thus by linear algebra:

$$l_{11}=1 \tag{8}$$

$$l_{21}=\rho_{21} \tag{9}$$

$$l_{22}=\sqrt{1-\rho_{21}^2} \tag{10}$$

$$l_{31}=\rho_{31} \tag{11}$$

$$l_{32}=\frac{\rho_{32}-\rho_{31}\rho_{21}}{\sqrt{1-\rho_{21}^2}} \tag{12}$$

$$l_{33}=\sqrt{1-\rho_{31}^2-\frac{(\rho_{32}-\rho_{31}\rho_{21})^2}{1-\rho_{21}^2}} \tag{13}$$

Thus the C matrix is factored into the lower triangle matrix L.

$$L=\begin{bmatrix} 1 & 0 & 0 \\ \rho_{21} & \sqrt{1-\rho_{21}^2} & 0 \\ \rho_{31} & \frac{\rho_{32}-\rho_{31}\rho_{21}}{\sqrt{1-\rho_{21}^2}} & \sqrt{1-\rho_{31}^2-\frac{(\rho_{32}-\rho_{31}\rho_{21})^2}{1-\rho_{21}^2}} \end{bmatrix} \tag{14}$$

L is postmultiplied by the independent random deviates to form correlated random deviates. It is the Cholesky decomposition that enables the Tecolote Risk Model to form correlated distributions of cost. In a two WBS element case, the correlation is done by correlating the second distribution to the first. Thus, the first distribution remains constant, while the second distribution is

transformed to a correlated (to the first distribution) distribution shape.

Tecolote Risk Model Verification With the Comparison Model

Overview. The Tecolote Risk Model computer program was not available for this research. Therefore, the same procedures as outlined above for the Tecolote Risk Model have been implemented in the Comparison Model. The Comparison Model is then used to verify the implementation of the Tecolote Risk Model.

Comparison Model Design. The Comparison Model was developed in Quattro Pro 3.0 using a 80386DX 20Mhz IBM AT compatible. The computer is equipped with 2 MByte of RAM and a 67 MByte hard disk. The model is limited to 2 WBS cost elements and triangular cost distributions. The model serves two useful purposes. The first is to compare the cost distributions before and after the Cholesky decomposition as the Tecolote Risk Model does not allow this visibility. Second, if the Tecolote Risk Model fails any test, then the same test can be applied to the Comparison Model to determine if the failure is with the methodology or the implementation.

Following is a step-by-step procedure of how the model was designed, including how the tests and other statistics were gathered. Book & Young and Jago are the primary sources of information in designing the Comparison Model (4:1-19; 12:1-12).

Independent Uniform Random Deviate Generation. Markland describes the process to generate the independent uniform random deviates from a pseudorandom number generator. The Comparison Model uses the multiplicative congruential method. The general form for this method is:

$$x_{n+1} = Kx_n(modulo\ m) \tag{15}$$

where K = 5^{13} = 1,220,703,125, m = $(2^{31} - 1)$ = 2,147,483,647. For the purposes of this model, x_n was chosen to be 10,000 (random seed 1). This pseudorandom number generator generates an independent string of random digits with a period of 2,147,483,647 (15:609-610). All cases have been tested with 4 other random number seeds. The seeds are 1,589,823,392 (random seed 2), 776,519,062 (random seed 3), 1,817,216,169 (random seed 4), and 641,504,206 (random seed 5). This was done to verify that the results are independent of the random number generator. All five seeds are statistically independent from each other. That is, the three lists of random deviates do not contain the exact same random deviate in any other list. The Comparison Model generates 2000 (1000 random draws for each of the two distributions) random numbers with a single random seed such that independence is guaranteed.

The random deviates are then divided by 2,147,483,647 to form 0-1 uniform random deviates. These are then used to generate the triangular random deviates needed to run the risk model.

Law and Kelton describe a triangular distribution generation method using 0-1 uniform random deviates. The following equation was used for this task:

$$\text{If } U \leq c, \text{ then } X = \sqrt{cU} \text{ else } X = 1-\sqrt{(1-c)(1-U)} \qquad (16)$$

where U represents the uniform random deviate draw and c is the mode of the triangular distribution. X represents a single Monte Carlo draw, which is replicated 1000 times for each distribution. The result is a 1 x 1000 vector for each cost element. This equation generates a 0-1 triangular distribution (14:261). To make the distributions closer to a real application, the 0-1 distribution is multiplied by a scalar value of 1000. Thus the Comparison Model generates two triangular distributions with a range of 0 to 1000.

The Comparison Model is designed to account for differences of scale using a standardization (normalization) technique. Book and Young describe the standardized Z-scores by the following equation:

$$Z_{j,k} = \frac{X_{j,k} - \mu_j}{\sigma_j} \qquad (17)$$

where j represents the WBS cost element and k is the Monte Carlo random deviate. The Z score is a standardized random deviate generated by the Monte Carlo method random number generator (4:7-8).

A note on the practical application of the standardization process is that it will maintain distributions in their proper proportion. That is if

distribution one has a range of 0 to 1000 with a mode of 750 and distribution two has a range of 0 to 100 with a mode of 75, then these two distributions could be highly correlated even though they have different ranges.

After the random deviates are generated and standardized, the next step is the Cholesky decomposition.

Cholesky Decomposition. The correlation matrix must be shown to be positive definite before continuing with the factorization. This research is limited to 2 WBS elements. All potential 2 x 2 correlation matrices are positive definite except when the correlation equals exactly -1 or 1; then the correlation matrix is positive semi-definite. A positive semi-definite matrix is a valid correlation matrix; however, it does not have a corresponding Cholesky factor. Therefore, this research must limit itself to positive definite matrices. The Cholesky decomposition correlation process only affects the 2nd of these two elements. In this research the second element is referred to as the non-pivot element.

The 2 x 2 correlation matrix, C, studied in the Comparison model is:

$$C=\begin{bmatrix} 1 & \rho_{21} \\ \rho_{21} & 1 \end{bmatrix}=\begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}\begin{bmatrix} l_{11} & l_{21} \\ 0 & l_{22} \end{bmatrix} \tag{18}$$

thus the matrix C is factored into the lower triangle matrix L and is multiplied by the standardized Z score vector to form the correlated Z^* score vector.

This is:

$$\begin{bmatrix} 1 & 0 \\ \rho_{21} & \sqrt{1-\rho_{21}^2} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ \rho_{21} z_1 + \sqrt{1-\rho_{21}^2}\, z_2 \end{bmatrix} = \begin{bmatrix} z_1^* \\ z_2^* \end{bmatrix} \tag{19}$$

The Comparison Model then uses the reverse standardization process to form the post factored distributions. This is accomplished by general methods as follows:

$$x_{jk}^* = \sigma_j Z_{jk}^* + \mu_j \tag{20}$$

The resulting x^*'s are correlated random Monte Carlo draws. The collection of 2000 (1000 for each distribution) of the two vectors forms the post factored distributions. Note again that distribution 1 is exactly the same as the pre-factored distributions and distribution 2 has changed depending on the correlation assigned between distribution 1 and 2.

The Comparison model then sums (convolution) the cost element distribution vectors. This forms the total cost distribution.

The Comparison Model uses Quattro Pro's frequency command to form a histogram (p.d.f.) of the cost elements and the total cost distributions. The p.d.f.s are then summed to form the c.d.f.s of each distribution.

One characteristic noted on Quattro Pro's frequency distribution is that if the interval reports 3 occurrences in the 75 to 100 interval, the 3 occurrences actually occur in the range 76 to 100. This has not been adjusted for,

since the model is for comparison uses only. If this model were to be used for an actual cost risk analysis, then the median of the intervals should be used. Since this model affects both pre and post factored distributions in the same manner, the affect is nullified and the test remains valid.

Other statistics that are recorded for each distribution pair are as follows: Total Cost Distribution's mean, variance, standard deviation, 1st non-zero point, and 1st 1000 point. The model also gathers statistics on the pre and post factored distribution 2 as follows: skewness, mode, 1st non-zero point, and 1st 1000 point. The 1st non-zero point and 1st 1000 point are used to define the range of the distributions.

Comparison Model Verification. Several tests were performed on the Comparison Model to verify that it was properly implemented. The first is a verification that the uniform independent random deviates are actually uniform. The second verification test is observing the shape of the distributions after the shape factors have been applied. The third test is verifying that the mean and variance are similar to the analytical solutions. The fourth test is verifying that the correlated random deviates are indeed correlated. The Comparison Model passed all four tests and this verified the methodology's implementation.

For Comparison Model verification, one test case was verified to have the user defined correlation. The post Cholesky decomposition random deviates were tested by using

SAS® PROC CORR (22:258-261) for one correlation value. Other correlations were tested visually by graphing the post Cholesky random deviates as an XY plot. The user defined correlations were indeed maintained for the one SAS® test case and the visual test over a range of correlations.

The Verification Process. The Comparison Model is used to verify the implementation of the Tecolote Risk Model. The mean, variance and end points of the total cost distributions were examined.

Validation of the Tecolote Risk Methodology

Overview. This section covers three topics for the Tecolote Risk Methodology validation process. First, the test data used to validate the methodology is described. Secondly, the selection of logically consistent correlations for the test cases (passing Failure Mode 1 from Figure 3) is made. Thirdly, the three criteria described in Chapter I (Failure Mode 2) are formally defined.

Data. The data that will be tested in this thesis are 25 pairs of triangular distributions. The triangular distribution is one of three possible types of distributions allowed in the Tecolote Risk Model (the Beta and Uniform are the other two types). There will be five test distributions in all. All five distributions range from 0 to 1000. The modes of the five distributions are: 0, 250, 500, 750 and 1000. The distributions will be tested against themselves and each other resulting in the 25 (5 x 5 = 25) test cases.

Since the total cost distribution is sensitive (Cholesky decomposition affect) to the order of lower level cost element distributions (4:17); both distribution 1 (Low 0 - Mode 250 - High 1000), distribution 2 (0-750-1000) and distribution 1 (0-750-1000), distribution 2 (0-250-1000) will be tested. It is important to remember that the first cost distribution is fixed, while the second distribution is altered. The correlation coefficient (ρ) will be allowed to vary from -0.9 to +0.9 in 0.1 increments for each of the 25 cases. The twenty-five cases are shown in Table 1.

Table 1 Test data

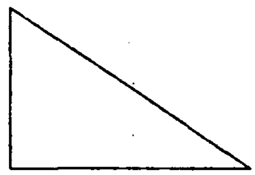
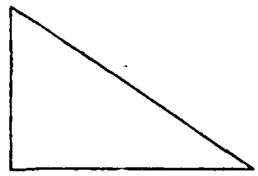
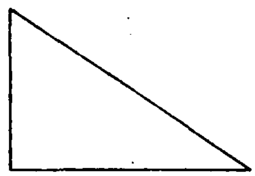
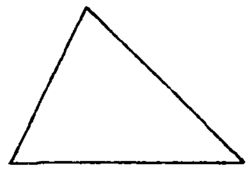
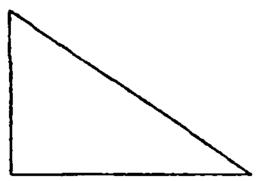
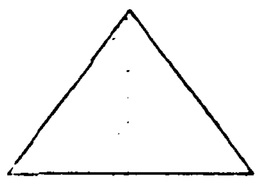
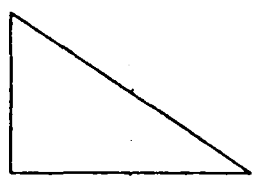
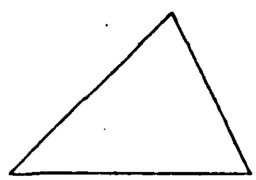
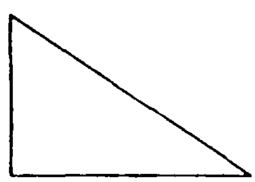
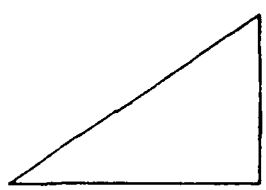
TEST CASE	DISTRIBUTION 1	DISTRIBUTION 2	CONSISTENCY TEST 1b CORRELATIONS
CASE 1	 MODE 0	 MODE 0	$0 \leq \rho < 1$
CASE 2	 MODE 0	 MODE 250	UNCERTAIN
CASE 3	 MODE 0	 MODE 500	UNCERTAIN
CASE 4	 MODE 0	 MODE 750	UNCERTAIN
CASE 5	 MODE 0	 MODE 1000	$-1 < \rho \leq 0$

Table 1 Test data continued

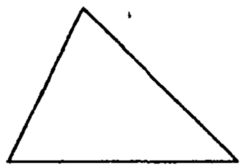
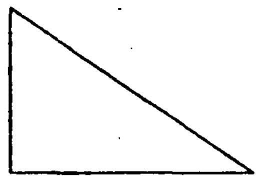
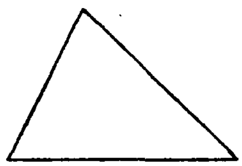
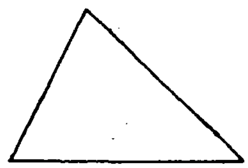
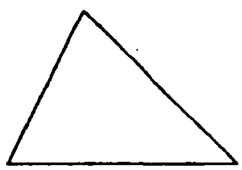
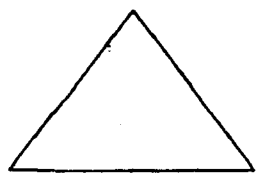
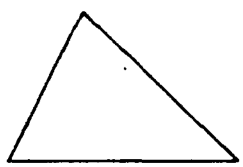
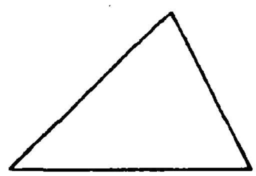
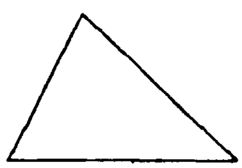
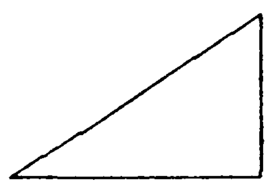
TEST CASE	DISTRIBUTION 1	DISTRIBUTION 2	CONSISTENCY TEST 1b CORRELATIONS
CASE 6	 MODE 250	 MODE 0	UNCERTAIN
CASE 7	 MODE 250	 MODE 250	$0 \le \rho < 1$
CASE 8	 MODE 250	 MODE 500	UNCERTAIN
CASE 9	 MODE 250	 MODE 750	$-1 < \rho \le 0$
CASE 10	 MODE 250	 MODE 1000	UNCERTAIN

Table 1 Test data continued

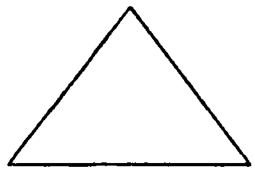
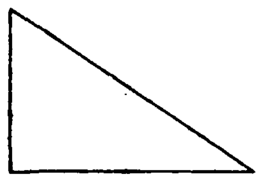
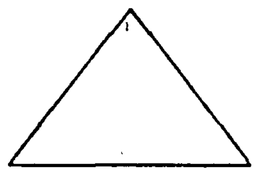
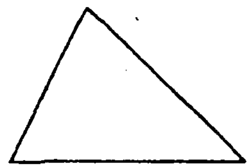
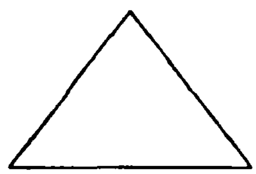
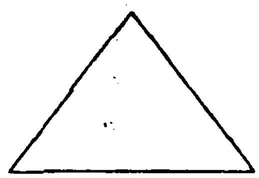
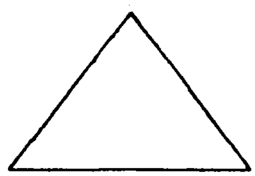
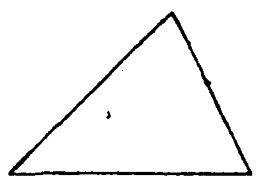
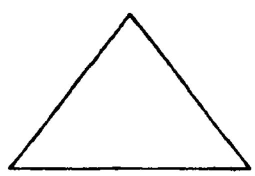
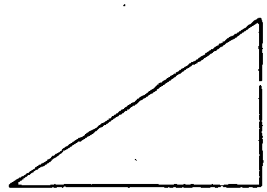
TEST CASE	DISTRIBUTION 1	DISTRIBUTION 2	CONSISTENCY TEST 1b CORRELATIONS
CASE 11	 MODE 500	 MODE 0	UNCERTAIN
CASE 12	 MODE 500	 MODE 250	UNCERTAIN
CASE 13	 MODE 500	 MODE 500	$-1 < \rho < 1$
CASE 14	 MODE 500	 MODE 750	UNCERTAIN
CASE 15	 MODE 500	 MODE 1000	UNCERTAIN

Table 1 Test data continued

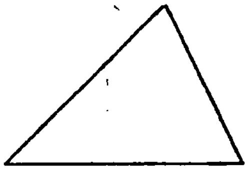
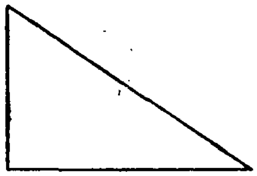
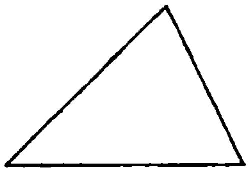
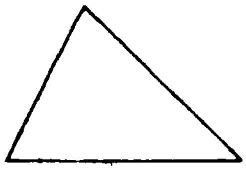
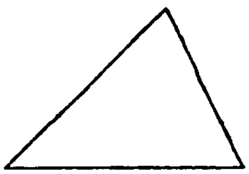
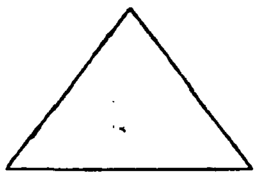
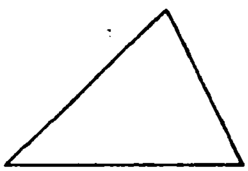
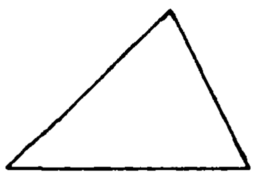
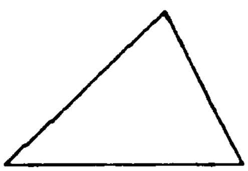
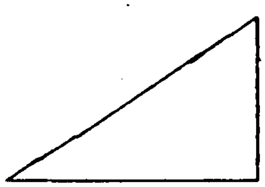
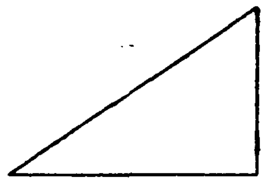
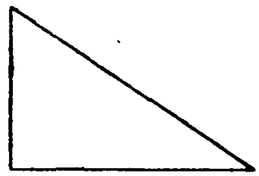
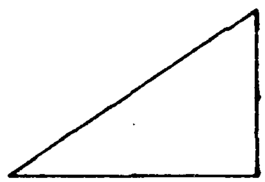
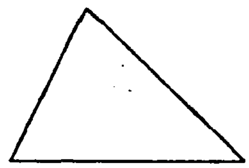
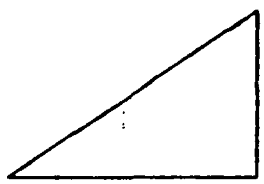
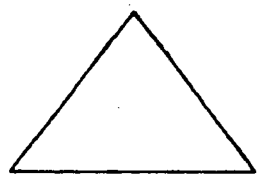
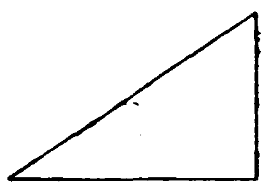
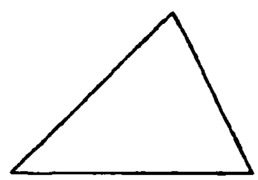
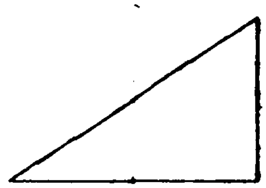
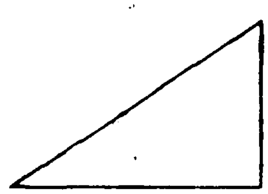
TEST CASE	DISTRIBUTION 1	DISTRIBUTION 2	CONSISTENCY TEST 1b CORRELATIONS
CASE 16	 MODE 750	 MODE 0	UNCERTAIN
CASE 17	 MODE 750	 MODE 250	$-1 < \rho \leq 0$
CASE 18	 MODE 750	 MODE 500	UNCERTAIN
CASE 19	 MODE 750	 MODE 750	$0 \leq \rho < 1$
CASE 20	 MODE 750	 MODE 1000	UNCERTAIN

Table 1 Test data continued

TEST CASE	DISTRIBUTION 1	DISTRIBUTION 2	CONSISTENCY TEST 1b CORRELATIONS
CASE 21	 MODE 1000	 MODE 0	$-1 < \rho \leq 0$
CASE 22	 MODE 1000	 MODE 250	UNCERTAIN
CASE 23	 MODE 1000	 MODE 500	UNCERTAIN
CASE 24	 MODE 1000	 MODE 750	UNCERTAIN
CASE 25	 MODE 1000	 MODE 1000	$0 \leq \rho < 1$

Consistent Input Parameters (User's Burden). The analyst must ensure that the input data is consistent. The user of any dependent cost element risk analysis methodology must specify internally consistent correlations. There are two restraints that the analyst must be concerned with. The first is that the correlation matrix must be positive semidefinite. The second is that the cost element distributions are logically consistent in relation to the properly specified correlations.

Although the only mathematical restriction on the correlation matrix is that it must be positive semidefinite, user's of the Tecolote Risk Model must test the correlation matrix for positive definiteness due to the use of the Cholesky decomposition. The test for positive definiteness is as described earlier in this chapter. It is straightforward and easily accomplished. This is referred to as Failure Mode 1a in Figure 3.

Logically consistent distributions in relation to specified correlations is a more intuitive exercise. This is referred to as Failure Mode 1b in Figure 3. According to Murphy, two distributions that are identically distributed can be independent or positively correlated. Any negative correlation between two identically distributed cost elements is illogical. The user should expect for two correlated identically distributed cost variables that if one cost element increases in cost then the second cost element should also increase in cost. The change in cost

should shift in the same direction. Of course if the cost elements are independent, then the direction of changes in cost between the two cost elements is not predictable. Two distributions that are opposed should be logically consistent for all negative correlations. Any positive correlation between opposed distributions should be logically inconsistent (18).

However, there is a much larger "gray" area of distributions, which does not have an obvious determination of consistency. There is uncertainty in what correlation range should exist between two non-identically distributed cost elements. If both cost elements are skewed right, but not identical, over what range may the correlation vary and still be consistent? This is a subjective question left to future research.

The selected test cases that are considered to be identically distributed will also be tested at +0.99 correlation. Cases that are considered to be opposed will be tested at -0.99. A -1 or +1 correlation coefficient cannot be tested because the correlation matrix is not positive definite.

The validation of the Tecolote Risk Methodology requires logically consistent inputs. The selected cases that are considered to be consistent are cases 1, 5, 7, 9, 13, 17, 19, 21 and 25 as displayed in Table 1. In addition, case 20 will be discussed and compared to the results of the other cases. Case 20 should have some positive correlation range

(absolute range is uncertain) since they are both left skewed distributions. Column 4 of Table 1 displays the correlations that are considered to pass the user's burden criteria from Figure 3. All distributions pass when the distributions are statistically independent; therefore 0 correlation is not noted in the table.

Validation Methodology (Methodology's Burden). Failure mode 2 requires the application of a set of tests or criteria on the cost risk methodology applied to properly specified input parameters. Criterion 2a is concerned with the input and output cost element correlations. Criterion 2b is concerned with the total cost distribution statistics. Criterion 2c is concerned with the cost element distribution shape.

Criterion 2a states that the user specified correlation matrix must be maintained through the cost risk methodology. To verify that the output correlations are equal to the input correlation, simply verify mathematically the output correlation. If the output correlation equals the user specified correlation, then the methodology is valid. The mathematical approach is the best validation process. However, by determining the correlation between the correlated distribution random deviates the user may also verify the methodologies' implementation. If the output correlations are equal to the user specified correlations, then the model passes this criterion.

Book and Young showed that the correlations specified by the user are maintained through the Cholesky decomposition (4:11).

Criterion 2b is the validation of the model's summary statistics for total cost. This criterion will use equations 3 and 4 described in Chapter I. This will be the validation of the methodology against the analytical solution. The mean and variance of the sum of distributions are easily computed.

Criterion 2c will test the change in shape of the second distribution as a function of the input correlation. This test will be accomplished with the use of the Chi-square goodness of fit test. Although all correlations are tested from -0.9 to 0.9, the only correlations that will be discussed in Chapter IV are those that are logically consistent.

The criterion 2c hypothesis test is:

H_0: The postfactored second WBS cost element distribution is equivalent to the user input second WBS cost element distribution

H_a: Reject H_0 if $\chi^2_{test} > \chi^2_{17df,\ 0.01}$

The Chi-square goodness of fit test uses the pre and post factored probability density functions (p.d.f.) from distribution 2. The p.d.f.s are divided into a total of 18 classification intervals. Sixteen of the intervals are of size 50 and the remaining two are -infinity to 100 and 900 to +infinity. The classification interval definition results in a 17 degrees of freedom Chi-square hypothesis

test. The Chi-square goodness of fit test is sensitive to the size of the classification interval (14:196-197). Therefore, the Chi-square test will also be tested with 10 and 34 classification intervals.

Newbold defines the Chi-square test as follows:

$$\chi^2 = \sum_{i=1}^{K} \frac{(O_i - E_i)^2}{E_i} \tag{21}$$

Where O_i is the observed frequency distribution, E_i is the expected frequency distribution and K is the number of intervals (20:414).

In the case of the Comparison Model, the expected frequency is the pre-factored distribution 2 and the observed frequency is the post-factored distribution 2.

In addition to the Chi-square test, this research will exhibit the $(O_i - E_i)^2/E_i$'s from the Chi-square test and the boundary charts (footprint) of the distributions as an analysis tool. The footprint or boundary graphs exhibit the maximum, minimum and mode of the pre and post factored second cost element distribution.

The Chi-square goodness of fit test could also be sensitive to the random number seed. Therefore, the random number seed was varied for the twenty-five test cases.

If the methodology passes criteria 2a, 2b and 2c, then the methodology is valid. It is possible that the methodology is valid only under certain conditions. These conditions will be described in Chapter IV.

Validity

The internal validity of this validation methodology is shown by the tests that were done. Extreme cases of distributions were tested. These are pairwise comparisons of the before and after factorization process. The testing methodology described in this chapter could be extended to multiple WBS cost elements. This paper limits the number of WBS elements to two for ease of analysis. That is, it is difficult to determine causality of a more complex WBS structure.

The Chi-square goodness of fit test is a commonly used statistical test for comparison of distribution shapes (20:412-413). This test should give the user some quantitative reason for limiting correlations given a set of cost element distribution shapes. Three interval sizes were evaluated to reduce Type I errors. A type I error is rejecting the null hypothesis when the null hypothesis should not be rejected. (20:332).

Analysis

The Chi-square goodness of fit statistic were plotted versus the correlation coefficient. If the distribution fails the H_0, then an investigation of why it failed must be made. The determination of where it failed was done with two other sets of data. The first is the footprint or boundary graph of the minimum, maximum and mode of the pre and post factored second distribution. The second is the

analysis of the $(O_i - E_i)^2/E_i$'s (referred to as interval statistic) for all the classification intervals in the Chi-square test.

Conclusions

The above methodology may be applied to any dependent cost risk analysis model. Specifically, this analysis will provide the Tecolote Risk model user with the ability to know the limitations of the cost risk model and verify implementation. The verification should be done in a sequence. First, verify the input parameters are internally valid and then determine if the input parameters are valid within the Tecolote Risk Model restrictions. If both conditions are met, then the analyst has some level of confidence that the input parameters are consistent.

IV. *Analysis*

Overview

This chapter applies the research methodology developed in Chapter III. All data (in graphical format) that was generated from the Comparison Model is included in the appendices. Cases 1 and 20 are reproduced here as well as in the appendices for clarity of discussion. The Tecolote Risk Methodology was tested against the Failure Mode 2 criteria as developed in Chapter III using valid input test parameters.

To reiterate, the validation of the Tecolote Risk Methodology is obtained either analytically or by simulating the result with the Comparison Model. The verification process refers to testing the Tecolote Risk Model (Air Force Risk Model, "riskmain.exe" dated 18 February 1991).

Methodology Validation

The Tecolote Risk Model source code was not available for this research; therefore other means had to be implemented in the validation process. The validation criteria are applied either mathematically or using the output of the Comparison Model. Validation criterion 2a is applied mathematically. Validation criteria 2b and 2c are applied through the Comparison Model.

Criterion 2a - Correlation Coefficient. Criterion 2a states that the user defined WBS cost element correlations

should be maintained through the cost risk model (i.e., input ρ = output ρ). In other words, the correlation between elements 1 and 2 should be ρ_{12}. Book and Young showed that the correlation between WBS cost elements is maintained through the Cholesky decomposition. From equation 19, the correlation between z_1^* and z_2^* may be verified:

$$CORR(z_1^*, z_2^*) = CORR(z_1, \rho_{21} z_1 + \sqrt{1-\rho_{21}^2} z_2) \tag{22}$$

$$= CORR(z_1, \rho_{12} z_1) + CORR(z_1, \sqrt{1-\rho_{12}^2} z_2)$$

$$= \rho_{12}\ CORR(z_1, z_1) + \sqrt{1-\rho_{12}^2} CORR(z_1, z_2) = \rho_{12}$$

CORR(z1,z1) equals 1 and, since the z scores are generated independently, CORR(z1,z2) equals 0. Equation 22 shows that the user defined correlation are maintained through the Cholesky decomposition. Book and Young include in their documentation that other pairs of cost elements maintain their correlation (4:11). The Tecolote Risk Methodology passes criterion 2a.

Criterion 2b - Mean and Variance. Criterion 2b states that the total cost mean and variance calculated by the cost risk model should be equal to the analytical total cost mean and variance resulting from equations 3 and 4.

Consider two triangular cost distributions. Both distributions have a range from 0 to 1000 with a mode of 250. The correlation is limited to three values: -0.5, 0,

and 0.5. The Trial column in Table 2 reflects the choice of the random number seed chosen for the pseudorandom number generator and these values are: 10,000 (random seed 1), 1,589,823,392 (random seed 2), 776,519,062 (random seed 3), 1,817,216,169 (random seed 4), and 641,504,206 (random seed 5). The five random number seeds generate independent random number strings for the simulation. Results of the simulation runs are shown in Table 2.

Table 2 Results of Tecolote Risk Methodology validation criterion 2b

CORRELATION	ANALYTICAL RESULT		TRIAL	COMPARISON MODEL RESULT	
	TOTAL COST MEAN	TOTAL COST VARIANCE		TOTAL COST MEAN	TOTAL COST VARIANCE
-0.5	833	45,139	1	847	46,320
			2	831	47,510
			3	840	44,707
			4	336	46,615
			5	831	44,789
0	833	90,278	1	847	92,474
			2	831	95,310
			3	840	89,220
			4	836	93,797
			5	831	89,753
0.5	833	135,417	1	847	138,963
			2	831	142,539
			3	840	133,804
			4	836	139,948
			5	831	134,380

Clearly, Table 2 exhibits that the total cost mean and variance properly reflect the analytical values calculated using equations 3 and 4. The Mann-Whitney non-parametric test for equivalent means was used to test the analytical mean against the simulation mean (5:224-229). The analytical values from equation 3 (see column 2) were compared to the simulation values from the Comparison Model (see column 5). The means from the two methods (analytical and simulation) are equivalent as tested by Mann-Whitney at the 90% confidence level. Equation 4 states that the total cost variance is equal to the sum of the cost element variances plus two times the covariance between the cost elements. Since covariance is a function of the correlation coefficient, the total cost variance should vary with correlation. By inspection the Tecolote Risk Methodology total cost variance (see column 6) reflects the total cost variance calculated analytically (see column 3) from equation 4. The Tecolote Risk Methodology passes this criterion. Other distributions and correlations were tested but are not included in this documentation. All other trials have the same result.

Criterion 2c - Distribution Shapes. Criterion 2c states that the input WBS cost element probability density function shapes should be the same as the output shapes. This chapter will describe the analysis for the 9 cases that have been assumed to be logically consistent. In addition case 20 will be described as an alternative case with an

uncertain range of logically valid correlations. Although 10 cases in all are discussed, the greatest detail will be on two cases (1 and 20).

An overview of the general findings is that the Tecolote Risk correlation methodology distorts the second cost distribution of the two WBS cost distributions defined. In fact, except for correlation values near 0 (independence), the post factored distribution is not equal to the user defined distribution. The correlation methodology is the multiplication of the independent distribution random deviates by the correlation matrix Cholesky factor (12:11). The affect on the distribution is as shown in Figure 7 and is indicated by three sets of statistics. The first is a Chi-square goodness of fit test between the pre and post factored second element distributions. The second is the change in the post factored distribution skewness. The third is the change in the range (upper and lower limits) and mode for the post factored distribution (this is referred to as the footprint or boundary of the distribution). Note that since the change in skewness and range are captured by the Chi-square goodness of fit test, the later two statistics will not be explicitly tested; they are simply a visual indication of the change in shape.

Note that since the range of the individual WBS element distributions are altered, so then is the total cost distribution. That is, if the user defines two cost distributions with a range of 0 to 1000 with any mode and

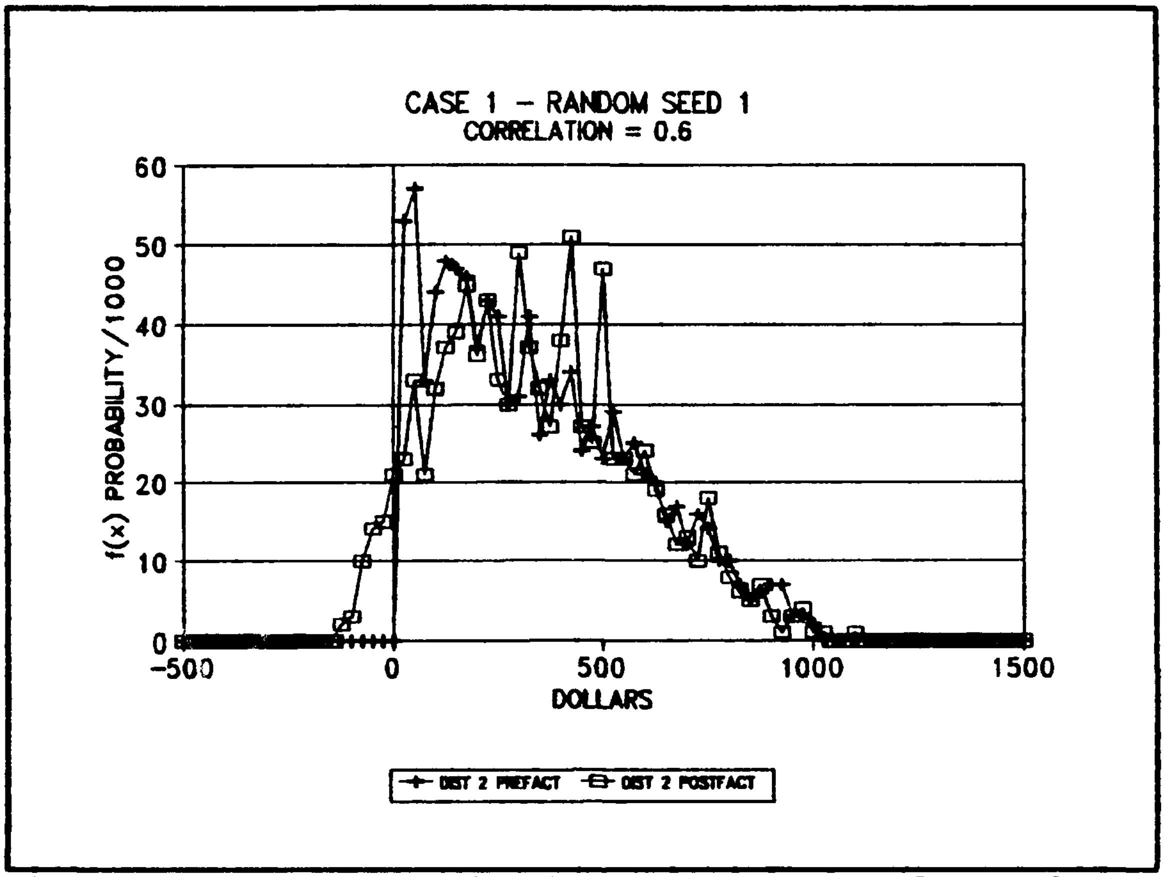


Figure 7 Case 1 - Distribution 2 pre and post factored cost probability density functions

any non-zero correlation, the total cost distribution will have a range from less than zero to greater than 2000. The user would expect the total cost distribution range to be from 0 to 2000.

The Chi-square goodness of fit graphs are the primary output from criterion 2c. To aid analysis of the Chi-square goodness of fit graphs, the boundary graph and a table of the interval statistics will be used. A description of how to use the boundary graph and interval statistics will be provided in the discussion of Case 1.

Case 1. Distributions 1 and 2 are identically distributed cost variables. Distribution 1 and 2 are defined over the range 0 to 1000, each with a mode of 0. Figure 8 displays the Chi-square goodness of fit test statistic and critical value over the range $-0.9 \le \rho \le 0.9$.

The goodness of fit test statistic is based on the difference between the pre and post factored second distributions. The goodness of fit test statistic quantifies what is visually seen in Figure 7. Figure 7 exhibits the pre and post factored distribution 2 from case 1 random seed 1. When $\rho = 0.6$, the largest interval statistic is in the interval 400 to 500. This can be seen in Figure 7 as well as in Table 3 ($\rho = 0.6$, interval 400 to 500, the interval statistic is 10). The mode of the distribution has changed considerably from the user defined value. Instead of being a right skewed right triangle, the post factored distribution is closer to being symmetrical. Figure 7 further indicates that the postfactored random deviates are being chosen outside of the user defined range. Three and one-half percent (35 observations / 1000 total observations * 100) of the postfactored distribution observations occur before the prefactored distribution minimum value. This means that the cost analyst has a negative cost 35 out of 1000 times for a logically consistent set of input parameters.

Figure 8 displays the 90% ($\alpha = 0.10$) and 99% ($\alpha = 0.01$) confidence level critical values for 17 degrees of freedom

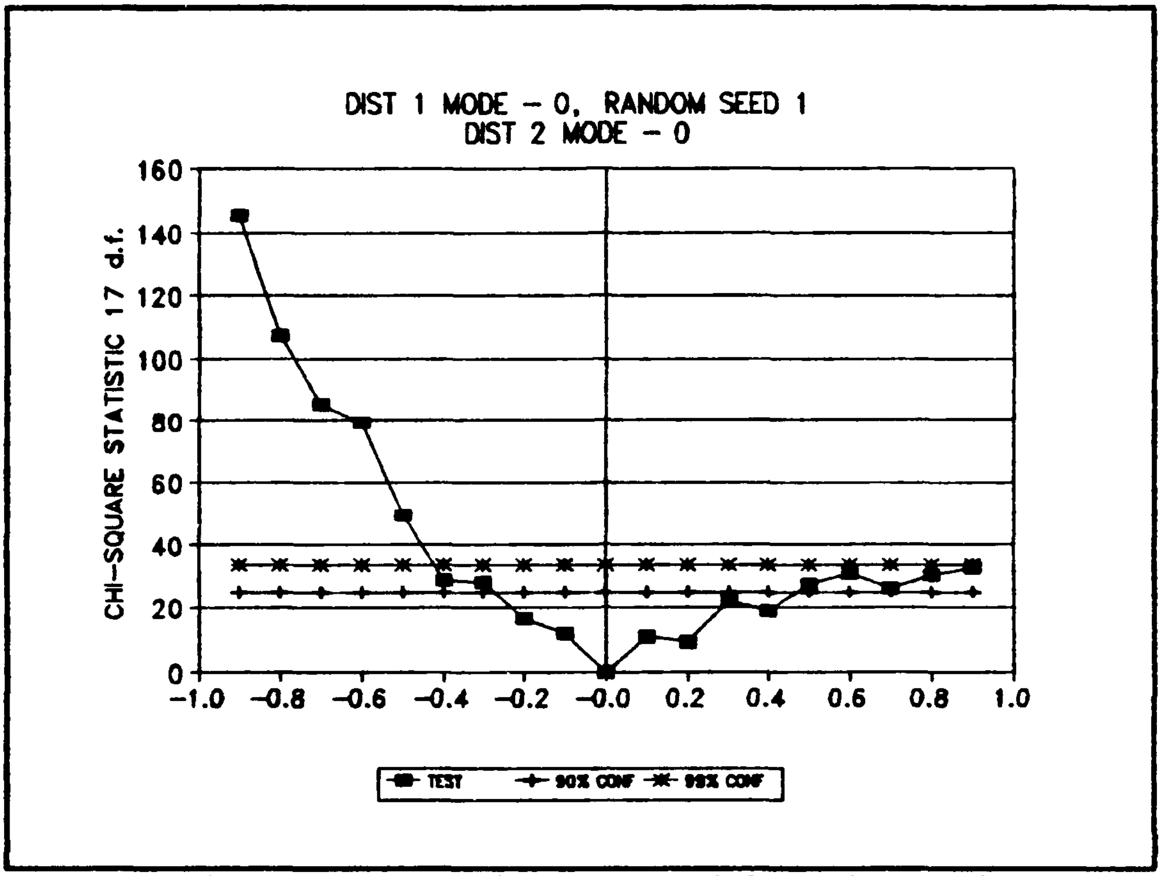


Figure 8 Chi-square test for Case 1 with random seed 1 and 17 d.f.

(d.f.). From Newbold, the critical values are 24.77 and 33.41 respectively. Any test statistic that exceeds the critical value fails the Chi-square goodness of fit test (20:412-416, 832-833).

When interpreting the Chi-square graphs, any correlation with a test statistic greater than the critical value fails the Chi-square goodness of fit test. Remember that Case 1 logical input correlations are limited to $0 \le \rho \le 0.9$. For example for the logical input parameters, the postfactored distribution fails the goodness of fit test at $\rho = 0.5$ at the 90% confidence level. This means that for correlations

ranging from 0 to 0.4, the postfactored distribution is equivalent to the prefactored distribution. At the 99% confidence level, the postfactored distribution never fails the goodness of fit test. The user should expect that the distribution pass for all positive correlations at either the 90% or 99% confidence level. In observing other Case 1 random seed trials, the maximum correlation value that passes for 99% confidence is at $\rho = 0.4$. Therefore in general the analyst should limit the correlation between Case 1 distributions to 0.4. The user should expect that the second distribution to fail the Chi-square test for negative correlations. The Chi-square test does indeed fail at the 90% confidence level at $\rho < -0.3$. The relative Chi-square test statistic is greater for negative correlations than it is for positive correlations for equally distant correlations from the origin. That is if the user compares the Chi-square statistic at -0.5 to the test statistic at 0.5, the Chi-square test statistic is larger for the logically inconsistent correlation definition.

According to Newbold, as the confidence level decreases, the confidence interval around the expected outcome decreases. As the confidence level varies, there is a tradeoff between Type I and Type II errors assuming everything else remains the same. Confidence level is equal to 1 - significance level (α). A Type I error (significance level or α) is the probability of rejecting a true null hypothesis. A Type II error (β) is the probability of

accepting a false null hypothesis. As the confidence level is increased (90% to 99%), the probability of a Type I error decreases. However, at the same time the probability of accepting a false null hypothesis increases (Type II error). Power is equal to 1 - ß. Power of a hypothesis test is correctly rejecting a false null hypothesis (20:329-335, 377-382). The Chi-square statistic remains constant for all confidence levels. However, the decision to accept or reject the null hypothesis is dependent on the user's acceptance of the confidence level - power tradeoff. The postfactored distribution may pass the Chi-square statistic at 99% confidence and fail at 90% confidence. The shape of the distribution or the Chi-square statistic does not change, only the acceptance or rejection of the null hypothesis changes.

Recall from Chapter III that the null hypothesis is that the postfactored second WBS cost element distribution is equivalent to the input second WBS cost element distribution. Therefore, if the confidence level is increased from 90% to 99%, the user decreases the probability of rejecting a null hypothesis when the he/she should not. The Chi-square hypothesis test confidence level (critical value) is chosen to be at 99%. This reduces the probability of a Type I error.

Two other measures ease the analysis of why the postfactored distribution is different than the input distribution. The two measures are boundary graphs and

interval statistic tables. The boundary graph illustrates when the Chi-square test is failing because the distribution is expanding beyond the original limits. The interval statistic table illustrates which interval has the largest difference between the prefactored distribution and the postfactored distribution.

Figure 9, Case 1 Boundary Chart, displays the maximum, minimum and mode for the pre and post factored second distribution. The solid box (■) indicates the prefactored distribution minimum value location, the asterisk (*) indicates prefactored maximum value location, the cross symbol (X) indicates the prefactored modal value location, the plus sign (+) indicates the postfactored minimum value location, the open box (□) indicates the maximum value location, and the filled triangle (▲) indicates the postfactored modal value location.

Figure 9 is related to Figure 7 by showing that the minimum, maximum and mode change as a function of correlation. Figure 7 indicates that the first observation from prefactored distribution 2 is at 25. The reader may confirm this with Figure 9 at correlation = 0.6 where the solid box is approximately 25. The same may be said for the maximum and modal values for the prefactored and postfactored distributions. Figure 7 indicates that the postfactored distribution minimum value is approximately -150, the same as Figure 9 for correlation = 0.6. The maximum value of the postfactored distribution is not so

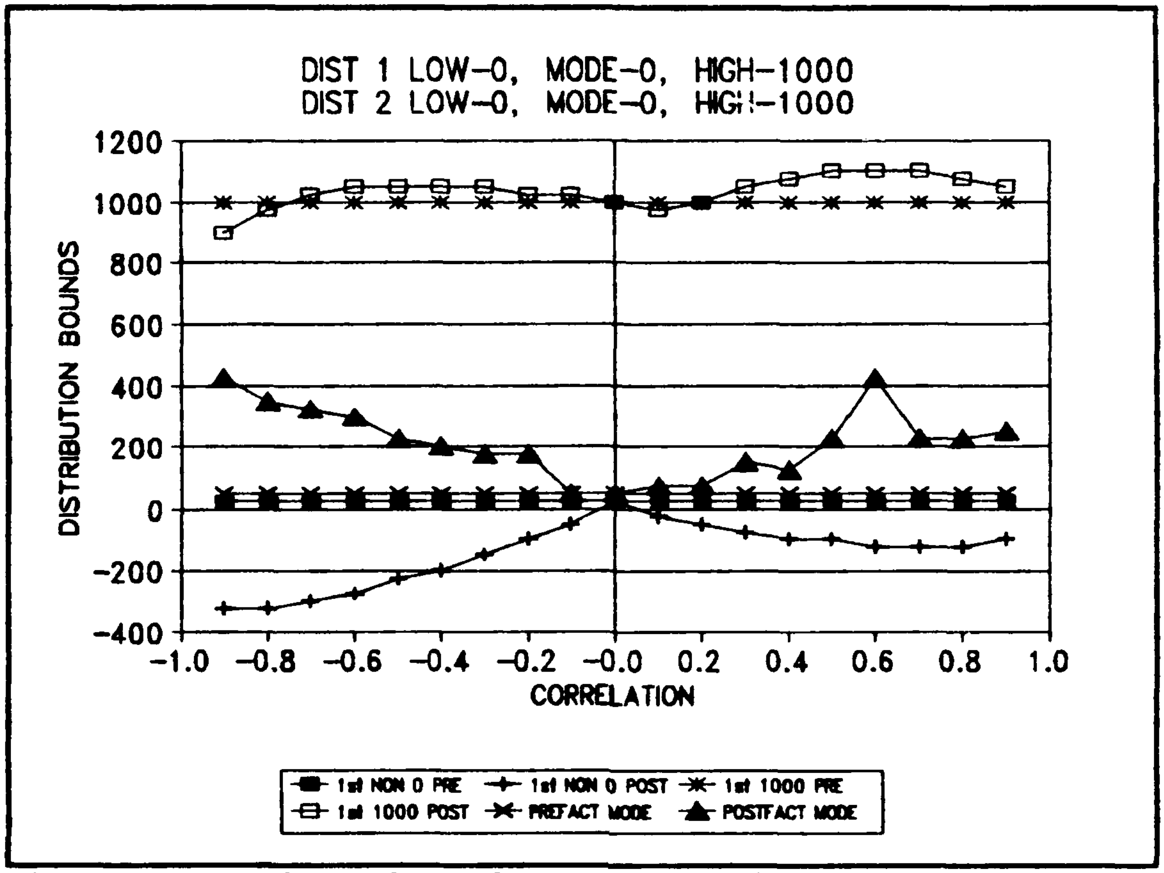


Figure 9 Boundary chart for Case 1 with random seed 1

easily seen in Figure 7 as it is in Figure 9. However, it is clear that there is at least one observation above 1000. Figure 9 shows that for correlation = 0.6, the maximum value is approximately 1100.

Note that all correlations ($-0.9 \le \rho \le 0.9$) are tested in Figure 8 and 9. The user should expect a larger Chi-square test statistic for logically inconsistent correlations. This is exhibited by the relatively larger test statistic for negative correlations than those calculated for positive correlations. The lower bound shown in Figure 9 indicates why the Chi-square test statistic is so large. The lower bound should be at 25 and for negative

correlations the lower bound ranges from approximately -350 to -50. The extension of the lower and upper boundaries affect the total cost distribution. If the user should specify Case 1 distributions with a -0.9 correlation, the total cost would range from -350 to whatever the maximum value is from distribution 1. Note that at correlation -0.9 the upper distribution bound is also decreased from the original limit. The total cost upper limit would be the maximum cost possible from distribution 1 plus approximately 900 from distribution 2. The postfactored mode moves toward the center of the distribution as correlation decreases (becomes more negative). The minimum value of the distribution varies as a function of correlation and even for logically consistent correlations, the lower bound decreases into the negative cost range. If the user defined a distribution with a lower limit of 0, the Tecolote Risk Model would actually draw negative costs from the distribution.

However, negative correlations are not logically consistent for Case 1 distributions. So the Tecolote Risk Model cannot be criticized for distorting the second distribution. The problem is that for logically consistent correlations, the upper and lower limits of the distribution are extended also. The distribution fails the Chi-square test for 90% confidence at $\rho = 0.5$. The boundary chart for Case 1 shows that the distribution lower limit is at -100.

The mode of the postfactored distribution also shifts as correlation varies. Figure 9 shows that the mode of the postfactored distribution $\rho = 0.6$ has shifted from 50 to 425. The interval statistics confirm that for Case 1 random seed 1 that this is indeed the case. This is specific to this case with random seed 1. There is a general trend of the mode to shift away from the prefactored distribution mode as the correlation is increased from zero. This general trend is true for all cases and random number seeds.

The interval statistics are in tabular format as shown in Table 3 for Case 1 random seed 1. The interval statistic, the $(O_i - E_i)^2/E_i$'s from each interval is used to evaluate where the distribution has been distorted the most for the evaluation cases. The $(O_i - E_i)^2/E_i$'s are the interval values that are summed to the Chi-square test statistic (20:414). This is an indication of where the distribution has changed the most in shape.

As exhibited in Figures 7, 8 and 9, the post factored distribution is distorted in test case 1. The valid correlations for this pair of distributions range from $0 \le \rho < 1$. For random seed 1, 17 degrees of freedom (d.f.), and 90% confidence level the distribution fails at $\rho = 0.5$ and never passes over the remaining range to $\rho = 0.9$. For the same test with 99% confidence, the distribution never fails over the same range. When investigating the $(O_i - E_i)^2/E_i$'s for each interval, the largest interval statistic is at the two lowest intervals of the distribution (i.e., -infinity to

Table 3 Case 1 - Random seed 1 interval statistics

Chi-sq		CORRELATION																		
INTERVAL		-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
-inf	100	0	0	2	3	2	2	1	0	0	0	0	0	0	1	1	1	1	0	0
100	150	18	22	15	14	15	8	3	1	1	0	2	0	0	1	2	4	8	14	4
150	200	11	8	5	1	0	2	2	0	1	0	0	2	1	0	0	0	0	0	2
200	250	8	3	2	0	3	0	0	1	0	0	0	1	0	1	1	1	0	0	1
250	300	0	0	9	32	12	3	0	4	2	0	0	1	7	2	5	5	3	3	9
300	350	0	15	17	0	0	3	5	2	0	0	0	1	0	1	1	0	0	0	0
350	400	5	8	1	6	2	0	0	1	0	0	0	0	0	0	0	0	2	2	1
400	450	38	11	8	5	2	0	0	0	0	0	1	0	1	1	4	7	3	1	0
450	500	16	6	7	3	2	2	3	0	1	0	0	0	0	1	3	10	2	2	1
500	550	6	8	0	0	0	2	1	0	0	0	2	0	3	2	0	1	0	0	0
550	600	10	1	2	4	2	0	1	0	0	0	2	0	3	3	2	0	0	0	0
600	650	3	1	3	3	1	2	3	0	0	0	0	0	0	0	1	0	1	2	1
650	700	1	6	3	2	2	1	1	0	1	0	1	0	1	0	4	1	2	0	8
700	750	6	7	4	1	0	1	3	3	3	0	0	1	4	0	1	0	0	0	1
750	800	2	3	1	2	2	1	4	3	1	0	0	0	0	0	0	0	0	3	0
800	850	0	0	0	1	0	1	0	0	0	0	1	0	1	2	0	0	0	0	1
850	900	8	3	5	2	3	1	0	0	1	0	0	1	0	2	0	1	2	1	0
900	+inf	15	5	1	1	0	0	0	0	0	0	0	1	1	0	1	1	1	1	3
Chi-sq	test	145	107	85	79	49	29	28	16	12	0	11	9	22	19	27	31	26	30	33
crt va	90%	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
crt va	99%	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33	33

100) for all correlations except one. The exception is at correlation = 0.6.

There does not seem to be a consistent pattern in where the interval statistics are the largest for a given random number seed and correlation. That is, as the correlation is varied, the Chi-square test statistic varies, but the interval that differs (prefactored distribution vs. postfactored distribution) the most varies.

When observing case 1 with the four other random number seeds, the distribution fails the goodness of fit test over a different correlation range. The ranges that the distribution passes the Chi-square test at 90% confidence for logical correlations and random seeds 2, 3, 4, and 5 are: $0 \le \rho \le 0.2$, $0 \le \rho \le 0.2$, $0 \le \rho \le 0.3$, and $0 \le \rho \le 0.4$. The results at 90% confidence and 99% confidence are summarized in Table 4.

Again the largest interval statistic is around the middle (around 500) of the distribution for mid-value correlations (0.4 to 0.7). This is true for 4 of the 5 random seed trials for case 1. The fifth trial has the largest interval statistic in the maximum interval. The Tecolote Risk Methodology will, with this pair of distributions, more heavily weight the center of the defined distribution than what it should. That is, more draws will come from around 500 than the user originally defined. This means that the Cholesky decomposition is affecting the mode.

Table 4 Range of acceptance for Case 1

CASE 1 - Range of acceptance assuming valid user definitions over the range $0 \le \rho < 1$, 17 d.f.		
	90% Confidence	99% Confidence
Random seed 1	$-0.2 \le \rho \le 0.4$	$-0.4 \le \rho \le 0.9$
Random seed 2	$-0.3 \le \rho \le 0.2$	$-0.3 \le \rho \le 0.3$
Random seed 3	$-0.1 \le \rho \le 0.2$	$-0.3 \le \rho \le 0.3$
Random seed 4	$-0.1 \le \rho \le 0.3$	$-0.2 \le \rho \le 0.4$
Random seed 5	$-0.3 \le \rho \le 0.4$	$-0.4 \le \rho \le 0.4$

The Chi-square goodness of fit test is sensitive to the random seed and to the Chi-square test classification interval size. The random number seed was varied for all test cases with 5 different random seeds and the classification interval size was varied over 3 values.

Table 4 indicates that the acceptance range varies as a function of the input random seed. This research investigated 5 random seeds for case 1 and case 20 to maintain manageability in the data set. The average acceptance range for all 5 different random number seeds is shown in Chapter IV for all 25 cases. As with all simulations, it is dangerous to make conclusions from a small number of replications (14:287). However, the data at hand does appear to be consistent. At the 90% confidence level, the maximum correlation value that is acceptable is $0.2 \le \rho \le 0.4$. At the 99% confidence level, with the exception of random seed 1, the maximum acceptable correlation is $0.3 \le \rho \le 0.4$. Random seed 1 would appear to be an outlier in this data set.

The second sensitivity area is the size of the test interval. Ten, 18, and 34 classification intervals were investigated. Law and Kelton state that interval sizing in Chi-square goodness of fit test is a difficult problem (14:196). The interval sizes are equal with the exception of the first and last. Law and Kelton state that equal interval sizes are not required. The power of the Chi-square goodness of fit statistic is dependent on the number

of classification intervals (14:196-197). The power of a hypothesis test refers to correctly rejecting the null hypothesis when the null hypothesis is false (20:332). The graphs for 10, 18 and 34 classification intervals were viewed and 18 was selected because it provided the best compromise between power and confidence. In general with 34 classification intervals the range of acceptable correlations decreased from those values depicted with the 18 classification intervals. Results from 10 classification intervals were similar to those from 34 classification intervals. With 10 classification intervals, the correlation acceptance range was narrower than with 18 classification intervals. Thus, 18 classification intervals is conservative in that it provides the Tecolote Risk Methodology with the greatest advantage.

The interval statistics are sensitive to the random number seed. For random seed 1, the interval statistic are evenly distributed except for three correlation values. For correlations 0.6 through 0.8, the largest distribution value starts in the middle and migrates to the minimum value. This is exhibited by Figure 9, the boundary chart for Case 1, in that the mode shifts at ρ = 0.6 and then the mode returns to a more smooth migration. The shift of the mode at ρ is due to the random number seed. The other random number seeds did not display this exact behavior in the mode. No strict conclusion may be made about the fact that the mode migrates as a function of correlation. Other

random number seeds migrate in different directions. The interval statistic only indicates where the distribution changes the most. There is no consistent interval where the distribution fails between random number seeds. Therefore, as a general evaluation tool, the interval statistic is of limited value. However, this does not infer that the Chi-square test statistic is invalid. The exact cost range that the distribution fails changes, but the distribution fails the Chi-square test irregardless of which random seed is chosen for mid to large correlations. The distribution fails in different quartiles depending on the random seed chosen.

As stated in the previous paragraph the largest interval statistic is dependent on the random number seed. This is further explained by the following: Random seed 2 has it's largest interval statistic in the 4th quartile of the distribution. Random seed 3 has the largest interval statistic around in the 2nd and 3rd quartiles of the distribution. Random seed 4 has it's largest Chi-square interval statistic in the 2nd and 3rd quartile of the distribution. Random seed 5 has the largest interval statistic in the 2nd quartile. Locating a single area of where the distribution fails to pass the Chi-square goodness of fit test is futile. The interval is too sensitive to the random number seed.

However, the overall Chi-square test statistic is relatively stable both in range and absolute value for different random number seeds.

The user of the Tecolote Risk Model (Air Force Risk Model) should limit the correlation for Case 1 to $-0.32 \leq \rho \leq 0.44$.

Case 20. Case 20 is defined with distribution 1 and 2 over the range 0 to 1000. Distribution 1's mode is 750 and distribution 2's mode is 1000. The logically consistent correlation range is uncertain for this pair of distributions.

Figure 10 shows how distribution 2 is distorted at correlation 0.5. With reference to Figure 10, distribution 2 is initially a left skewed right triangle and at correlation = 0.5, the postfactored distribution's mode has shifted left. Three and three-thirds percent (3.3%) of the postfactored distribution is greater than the bound for the prefactored distribution at correlation = 0.5. This is as shown in Figure 10. The interval statistic was investigated the same way as for Case 1. The interval statistic does not behave consistently across different random number seeds. Therefore, as an analysis tool, it is of little use except for pointing out exactly where the largest change in the distribution occurred.

It is clear that the Chi-square statistic, Figure 11, itself initially increases as correlation increases from 0

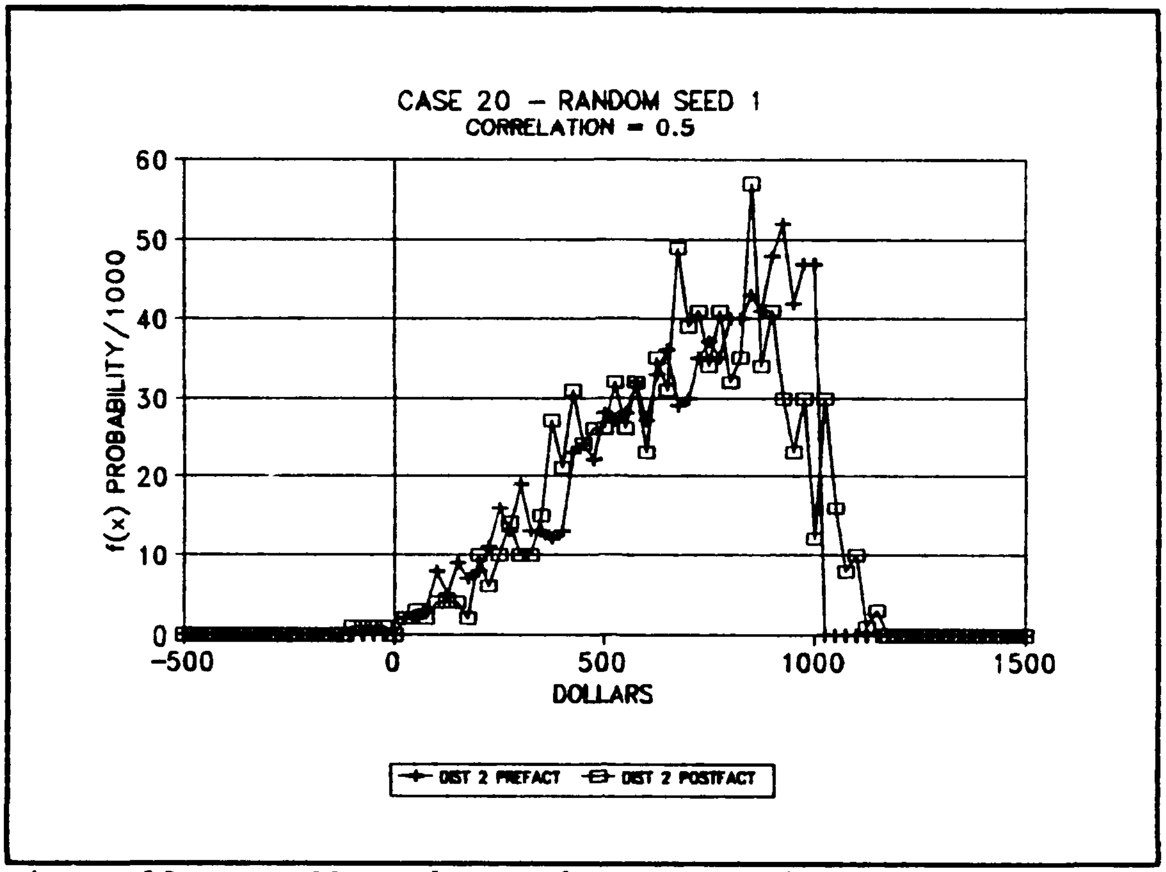


Figure 10 Case 20 random seed 1 - pre and post factored cost probability density functions

to 0.5 and then remains relatively constant. This is also true for Case 20 for the other four random number seeds as well.

Figure 12 shows how the distribution boundaries increase as the correlation moves away from independence. This supports the large Chi-square test statistic for negative correlations and larger positive correlations.

Table 5 indicates that there is greater consistency in Case 20 between random number seeds than in Case 1. The random number seeds are the same for both cases. No offer of an explanation is made in the regard of random number

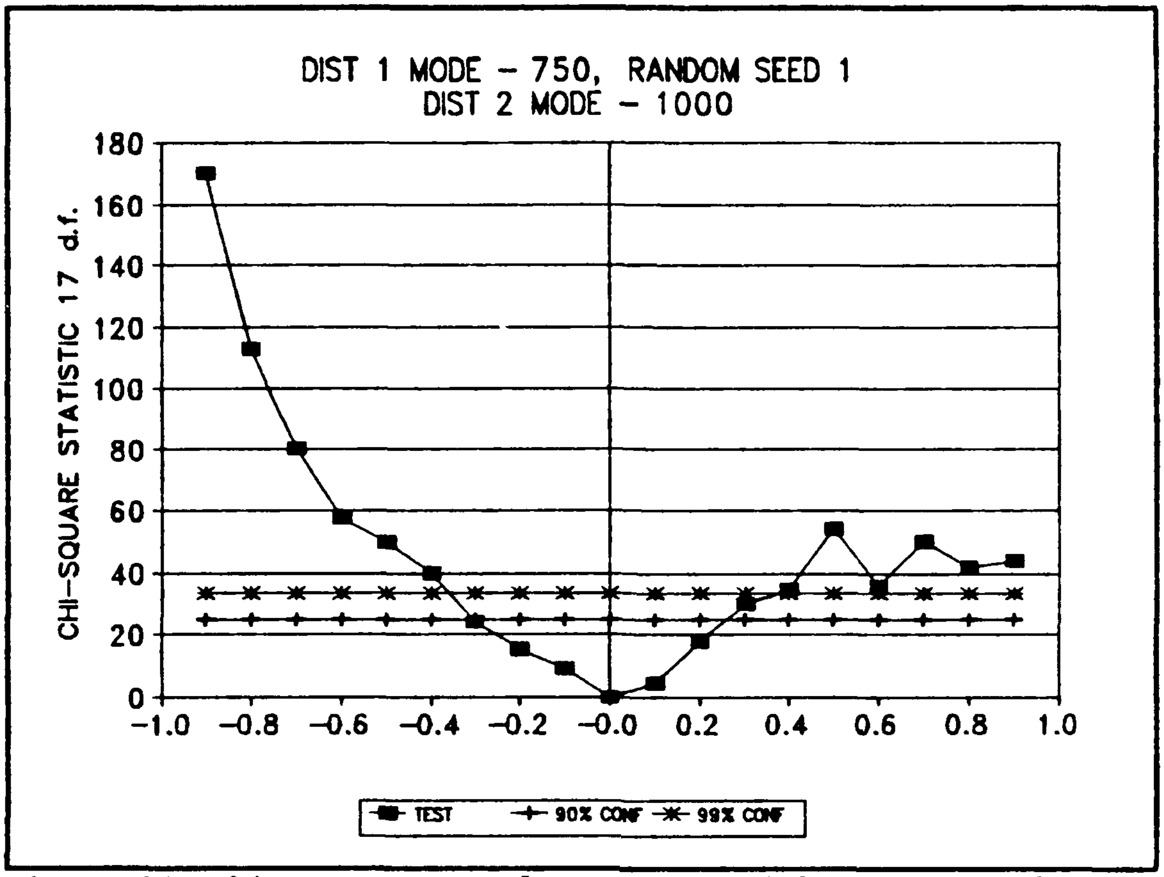


Figure 11 Chi-square test for Case 20 with random seed 1 and 17 d.f.

seeds affecting the result between different cases. However, the number of replications made for this research could be expanded to increase the data set and fidelity in the conclusions.

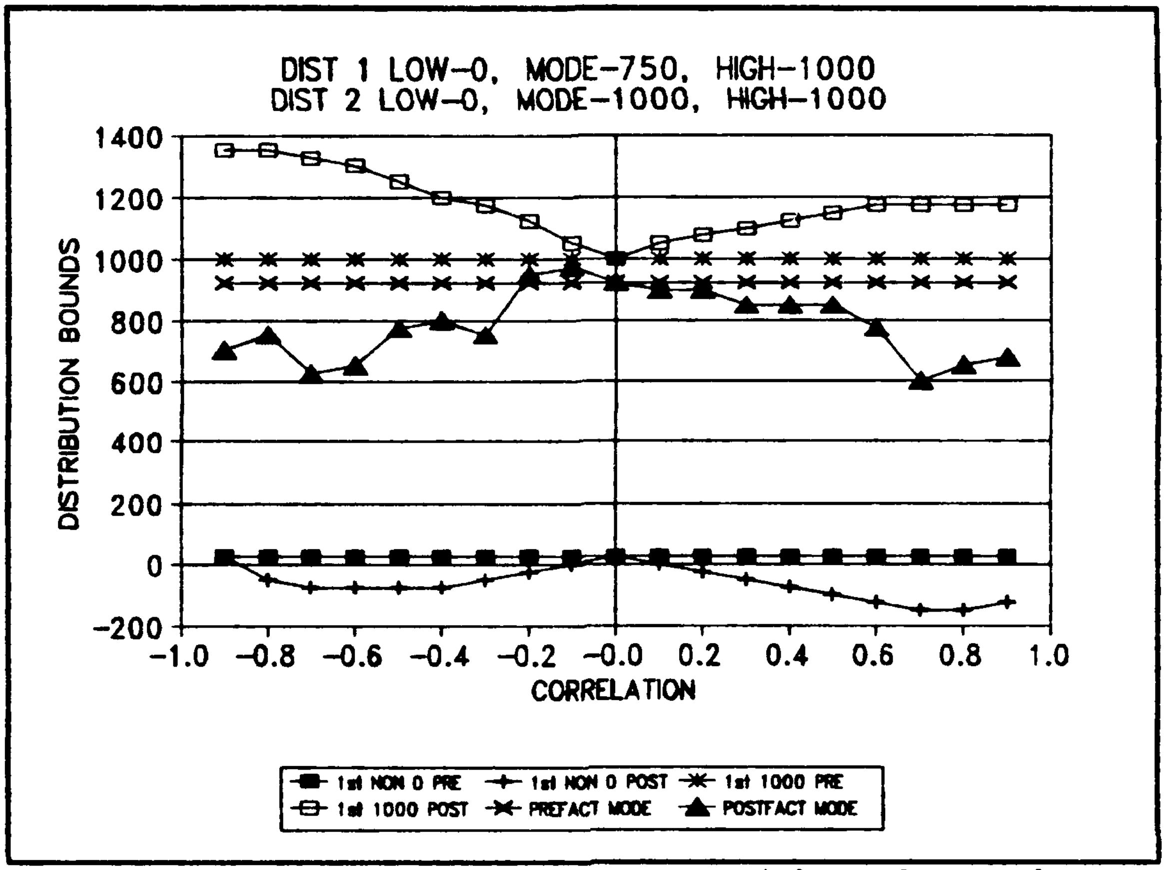


Figure 12 Boundary chart for Case 20 with random seed 1

Table 5 Range of acceptance for Case 20

CASE 20 - Range of acceptance is unknown - the range $-0.9 < \rho < 0.9$ will be considered		
	90% Confidence	99% Confidence
Random seed 1	$-0.3 \leq \rho \leq 0.2$	$-0.3 \leq \rho \leq 0.3$
Random seed 2	$-0.3 \leq \rho \leq 0.3$	$-0.4 \leq \rho \leq 0.3$
Random seed 3	$-0.3 \leq \rho \leq 0.3$	$-0.5 \leq \rho \leq 0.3$
Random seed 4	$-0.1 \leq \rho \leq 0.3$	$-0.2 \leq \rho \leq 0.3$
Random seed 5	$-0.1 \leq \rho \leq 0.3$	$-0.3 \leq \rho \leq 0.3$

The user of the Tecolote Risk Model (Air Force Risk Model) should limit the correlation between Case 20 type distributions to $-0.34 \le \rho \le 0.3$.

Other Cases. Cases 5, 7, 9, 13, 17, 19, 21 and 25 will be discussed in summary terms as well as all 25 cases. The range of correlation logical consistency (column 4) for the cases are as shown in Table 6. Table 6, columns 2 and 3 indicates the range of correlation acceptance for all 25 cases (results averaged for 5 random seeds). The Air Force Risk Model user should limit the correlation between pairs of distributions to the values shown in column 3 (99% confidence).

Table 6 The average acceptance range for all 25 cases

The average of 5 random number seed acceptance ranges for all 25 cases at 90% and 99% confidence levels			
	90% Confidence	99% Confidence	Logical consistency range
CASE 1	$-0.20 \le \rho \le 0.30$	$-0.32 \le \rho \le 0.44$	$0 \le \rho < 1$
CASE 2	$-0.20 \le \rho \le 0.20$	$-0.30 \le \rho \le 0.36$	UNCERTAIN
CASE 3	$-0.30 \le \rho \le 0.44$	$-0.54 \le \rho \le 0.50$	UNCERTAIN
CASE 4	$-0.32 \le \rho \le 0.20$	$-0.40 \le \rho \le 0.26$	UNCERTAIN
CASE 5	$-0.34 \le \rho \le 0.22$	$-0.42 \le \rho \le 0.28$	$-1 < \rho \le 0$
CASE 6	$-0.22 \le \rho \le 0.30$	$-0.32 \le \rho \le 0.46$	UNCERTAIN
CASE 7	$-0.20 \le \rho \le 0.24$	$-0.28 \le \rho \le 0.34$	$0 \le \rho < 1$
CASE 8	$-0.36 \le \rho \le 0.40$	$-0.48 \le \rho \le 0.56$	UNCERTAIN
CASE 9	$-0.28 \le \rho \le 0.18$	$-0.36 \le \rho \le 0.24$	$-1 < \rho \le 0$
CASE 10	$-0.26 \le \rho \le 0.24$	$-0.44 \le \rho \le 0.34$	UNCERTAIN
CASE 11	$-0.22 \le \rho \le 0.30$	$-0.38 \le \rho \le 0.42$	UNCERTAIN
CASE 12	$-0.24 \le \rho \le 0.24$	$-0.28 \le \rho \le 0.38$	UNCERTAIN

Table 6 Range of acceptance for 25 cases continued

The average of 5 random number seed acceptance ranges for all 25 cases at 90% and 99% confidence levels			
	90% Confidence	99% Confidence	Logical consistency range
CASE 13	$-0.48 \le \rho \le 0.56$	$-0.64 \le \rho \le 0.78$	$-1 < \rho < 1$
CASE 14	$-0.26 \le \rho \le 0.22$	$-0.32 \le \rho \le 0.28$	UNCERTAIN
CASE 15	$-0.26 \le \rho \le 0.26$	$-0.40 \le \rho \le 0.36$	UNCERTAIN
CASE 16	$-0.28 \le \rho \le 0.30$	$-0.38 \le \rho \le 0.40$	UNCERTAIN
CASE 17	$-0.20 \le \rho \le 0.20$	$-0.30 \le \rho \le 0.32$	$-1 < \rho \le 0$
CASE 18	$-0.42 \le \rho \le 0.46$	$-0.52 \le \rho \le 0.70$	UNCERTAIN
CASE 19	$-0.14 \le \rho \le 0.26$	$-0.30 \le \rho \le 0.32$	$0 \le \rho < 1$
CASE 20	$-0.22 \le \rho \le 0.24$	$-0.34 \le \rho \le 0.30$	UNCERTAIN
CASE 21	$-0.22 \le \rho \le 0.22$	$-0.36 \le \rho \le 0.38$	$-1 < \rho \le 0$
CASE 22	$-0.22 \le \rho \le 0.26$	$-0.32 \le \rho \le 0.28$	UNCERTAIN
CASE 23	$-0.40 \le \rho \le 0.54$	$-0.50 \le \rho \le 0.68$	UNCERTAIN
CASE 24	$-0.20 \le \rho \le 0.26$	$-0.40 \le \rho \le 0.40$	UNCERTAIN
CASE 25	$-0.18 \le \rho \le 0.24$	$-0.30 \le \rho \le 0.38$	$0 \le \rho < 1$

Cases 3, 8, 13, 18, and 23 which defined the symmetrical distribution for the second distribution had the largest correlation range of acceptance. Specifically, Case 13 had the widest range of acceptance of all cases tested. Case 13 is as expected since the logical correlation range for it is $-1 < \rho < 1$. Cases 11, 12, 14 and 15 which had the symmetrical distribution for the first distribution did not have the same advantage in acceptance range.

Case 13 is interesting because both distributions are symmetrical. The Cholesky decomposition is suggested by

Johnson as a random deviate correlation method for normal variates (13:52-55). Although the symmetrical triangular distribution is not normal, it does infer that symmetrical distributions may be able to utilize the Cholesky decomposition to correlate random deviates over a wide range of correlations.

Tecolote Risk Model Verification

Of the three criteria described in Chapter III, only 2b can be used to verify the implementation of the Tecolote Risk Model. Criterion 2a has been accomplished mathematically. However, it remains unknown if the Cholesky factor has been applied correctly in the Air Force Risk Model. An investigation of the Air Force Risk Model computer program would be necessary. Criterion 2c would require access to the model's random deviates. The Tecolote Risk Model does not allow access to the random deviates. Thus, this research can only investigate the difference between the simulation and analytical total cost summary statistics (criterion 2b).

Verification Criterion 2b. The verification of the Tecolote Risk Model (Air Force Risk Model) has been done with the "Riskmain.exe" file dated 18 February 91. The random number seed cannot be controlled in this version of the Tecolote Risk Model. Therefore, the random number seed is both unknown and non-repeatable. The input parameters are the same as used for the validation process and are two

triangular cost distributions with a range of 0 to 1000 with a mode of 250. The correlation is varied over three values. Other trials with different distributions and correlations were tested with the results being the same. Table 6 shows the result of the Tecolote Risk Model verification.

Table 7 Results of Tecolote Risk Model verification criterion 2b

CORRELATION	ANALYTICAL RESULT		TRIAL	TECOLOTE RISK MODEL RISKMAIN.EXE DATED 18 FEB 91	
	TOTAL COST MEAN	TOTAL COST VARIANCE		TOTAL COST MEAN	TOTAL COST VARIANCE
-0.5	833	45,139	1	850	88,578
			2	825	85,270
			3	836	91,041
			4	826	82,042
			5	849	88,500
0	833	90,278	1	812	87,243
			2	823	89,467
			3	832	86,013
			4	830	88,881
			5	833	79,738
0.5	833	135,417	1	836	88,923
			2	823	84,187
			3	828	84,048
			4	817	88,703
			5	816	84,856

Clearly Table 7 results show that the total cost mean behaves as would be expected from equation 3. The analytical mean calculated using equation 3 (see column 2) is roughly equivalent to the mean from the Tecolote Risk Model (see column 5). The Mann-Whitney test for equivalent means was used to verify the equivalence of the simulation mean and the analytical mean (5:224-229). The test showed that at the 90% confidence level the sum of the means from the analytical solution is equal to the simulation mean. However, the total cost variance (see column 3) does not reflect what would be expected from equation 4 (see column 6). For example, at correlation = -0.5, the variance should be 45,139. The Tecolote Risk Model generates variances between 82,042 and 91,041. At ρ = 0.5, the same problem is exhibited. The total cost variance appears to be unaffected by the correlation coefficient since the average for correlations -0.5, 0, and 0.5 are respectively: 81,686, 86,268, and 86,143. The total cost variance has a general tendency for the case of independence. Equation 4 states that the total cost variance is equal to the sum of the cost element variances plus two times the covariance between the lower level cost elements. Since covariance is a function of the correlation coefficient, the total cost variance should vary with correlation. The Tecolote Risk Model (Air Force Risk Model; "riskmain.exe dated 18 February 1991) does not pass this verification criteria.

Internal Validity

The results for criterion 2b were tested for sensitivity to random number seed. The results are that they are not sensitive to the random number seed.

Sensitivity analysis for Chi-square test classification interval size and to random number seed were performed for criterion 2c. The result is that the test is sensitive to both parameters. The general trend is that the results are valid in a broad perspective. That is, if the user limits the correlation coefficient to low values for logically shaped distributions, the total cost distribution will probably be valid.

V. *Conclusion and Recommendations*

Conclusions

Air Force Risk Model. The Air Force Risk Model (Tecolote Risk Model, "riskmain.exe" file dated 18 February 1991) is not a valid implementation of the Tecolote Risk Methodology. The total cost distribution variance does not correspond to the analytically determined values. Tecolote Research, Inc. was notified of this problem on 28 May 1991 and they have located a software problem.

Tecolote Risk Methodology. Once the user has defined logically consistent input parameters the risk methodology can be tested. This has been accomplished in this research for triangular distributions. Criteria 2a, 2b and 2c were used to evaluate the Tecolote Cost Risk Methodology.

Criterion 2a, states that the user defined component correlations should be maintained through the cost risk model (i.e., input ρ = output ρ). This has been shown by Book and Young mathematically to be true for the Tecolote Risk Methodology (4:11). Therefore, the Tecolote Risk Methodology satisfies criterion 2a.

Criterion 2b, states that the total cost mean and variance calculated by the cost risk model should be equal to the analytical total cost mean and variance. This has been shown to be the case through the simulation. Therefore, the Tecolote Risk Methodology satisfies criterion 2b.

Criterion 2c, states that the input WBS cost element probability density function shapes should be the same as the output shapes. This has been tested with the Chi-square goodness of test. Twenty-five cases of triangular distribution pairs with the correlation coefficient varied from -0.9 to 0.9 were tested. Several of these cases were identified as consistent input parameters and therefore serve to test the hypothesis. The Tecolote Risk Methodology satisfies criterion 2c under limited conditions. There is a narrow range of acceptable correlations allowed to be input to the model. The cost analyst should not use a correlation greater than 0.4 (-0.4) for distributions that are assumed to have positive (negative) logical correlation.

The Tecolote Risk Methodology is valid under tight constraints. The Chi-square goodness of fit test indicates that the model is distorting the user defined cost distributions. This author recommends the usage of the 18 classification interval (17 degrees of freedom) for the determination of where the cost distributions are not distorted. There is a difference in the Chi-square test statistic for 10, 18 and 34 classification intervals. Ten and 18 classification intervals provide tighter constraints for valid correlation input parameters given a distribution shape.

The Tecolote Risk Model allows input of triangular, beta, and uniform distributions only triangular distribution were explicitly tested. However, an extrapolation from this

data set may be inferred to the beta distribution. Both the triangular and beta distribution have finite limits, multiple skewness coefficients, and a single mode (the beta distribution is more flexible in that variance may also be varied). The analyst may assume that the beta distribution will be distorted similarly to the triangular distribution.

Recommendations

There are two types of recommendations. The first recommendation type is how the user should apply the Air Force Risk Model and the second is recommendations to the Air Force Risk Model developer.

The cost analyst should limit the use of the Air Force Risk Model (Tecolote Risk Model) to relatively small correlations. More specifically, the user should limit the correlations to the values shown in Table 6.

The cost analyst should calculate the analytical total cost mean and variance as a cross-check for the Tecolote Risk Model.

The remainder of this section is to the Air Force Risk Model developer. The implementation should be modified to include the seed and display the output. The user should be able to input the random seed number for any risk analysis "run". This allows repeatability of the simulation to make sensitivity analysis less difficult. Since all simulation studies should be based on multiple replications, the model should have the capability to calculate the average of

independent replications. This could be offered as an option in the menu tree.

The Air Force Risk Model should display both the probability distribution function and the cumulative distribution function at every level in the WBS structure. The order that the mean, mode and standard deviation are displayed on the graphs in the model should be consistent.

Further Research. An investigation into alternative $C = LL^T$ factorization algorithms should be made. The Cholesky decomposition produces a repeatable correlation matrix factor. However, the L factor is not unique. Other factors exist, and these may not distort the cost distributions as much as Cholesky decomposition. A similar test to the one accomplished in this research may be replicated for other factorization algorithms.

An investigation of the affect the other three uncertainty types (schedule, technology, and configuration) allowed in the Air Force Risk Model have on cost.

An investigation into the Cholesky decomposition affect on the beta and uniform distribution is recommended. The Air Force Risk Model accepts input of beta, triangular and uniform cost distributions. The same criteria used in this research could be used for the beta and uniform distributions.

An investigation of the affect that Cholesky decomposition has on the "ith" cost element. Assuming cost

dependencies, how is the "10th" WBS cost element affected by the previous 9 cost elements?

Research should be done on how to identify the proper correlations between WBS elements. That is, which data should be used (T1s, total cost, down some learning curve, etc).

Does the correlation matrix change as a function of program maturity? That is, one might expect a dense correlation matrix for new programs and a sparse correlation matrix for mature programs.

If a correlation matrix is not positive definite, there is no current method to identify the pair(s) of WBS elements that are not consistent.

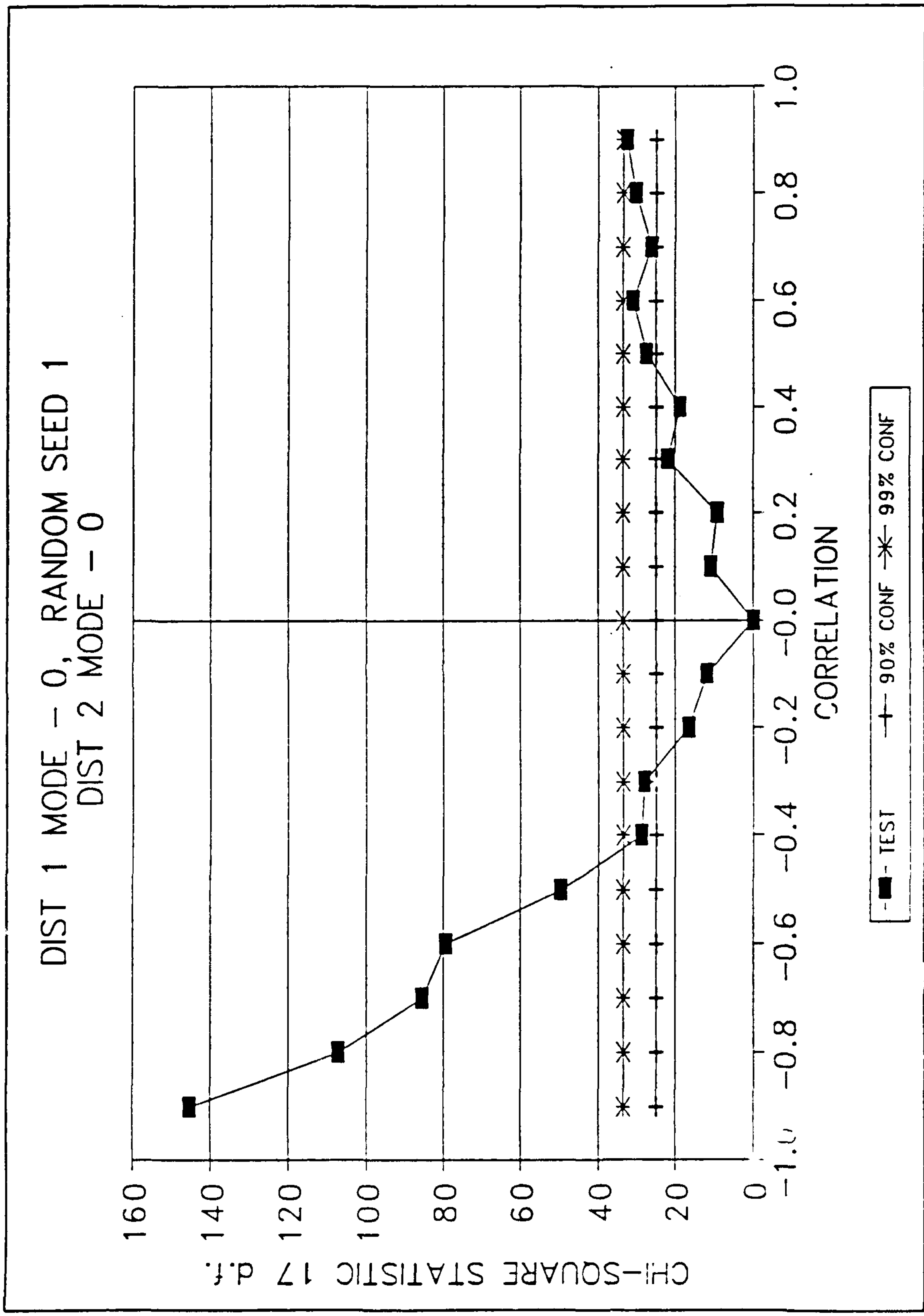


Figure 13 Chi-square test for Case 1 with random seed 1 and 17 d.f.

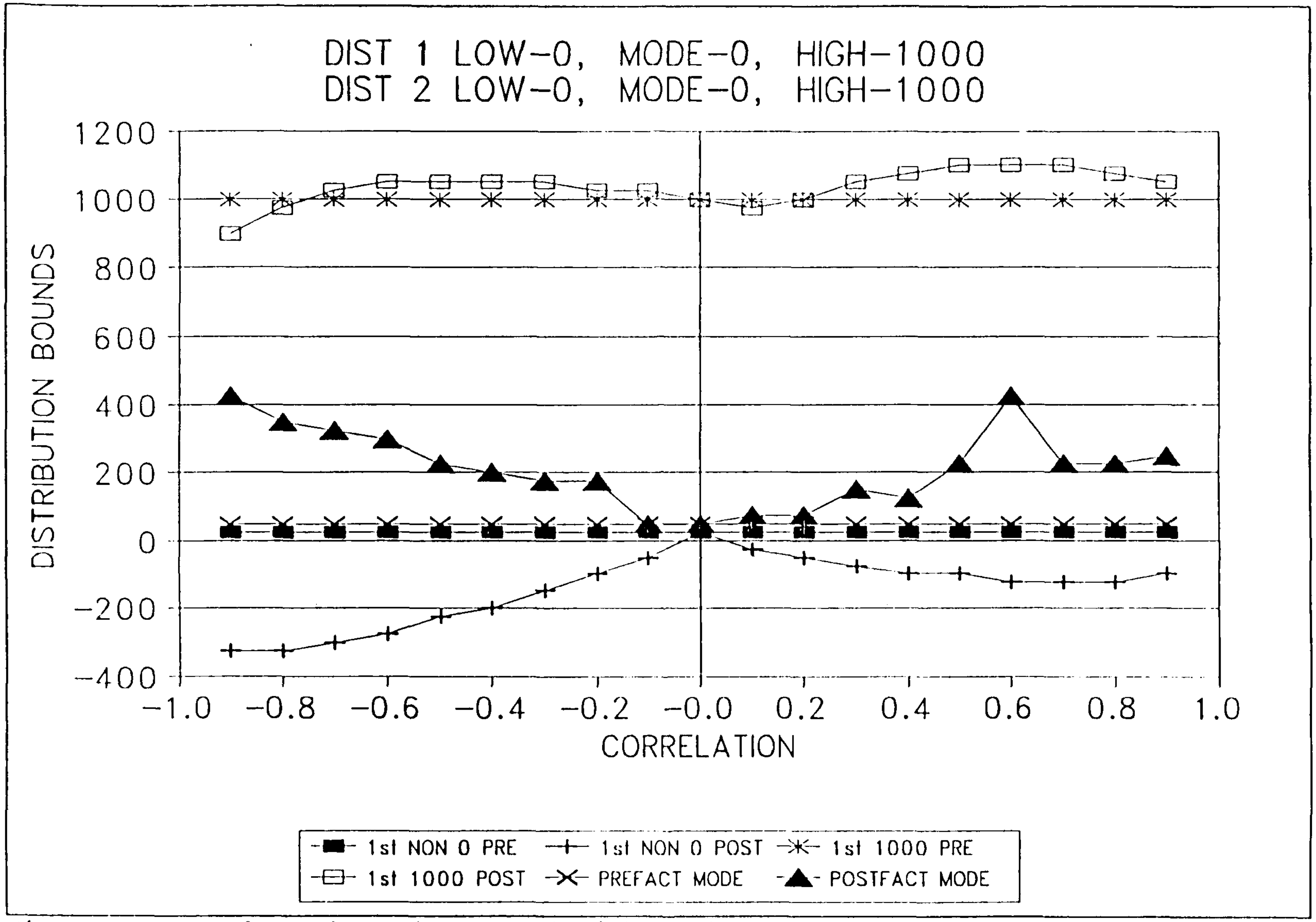


Figure 14 Boundary chart for Case 1 with random seed 1

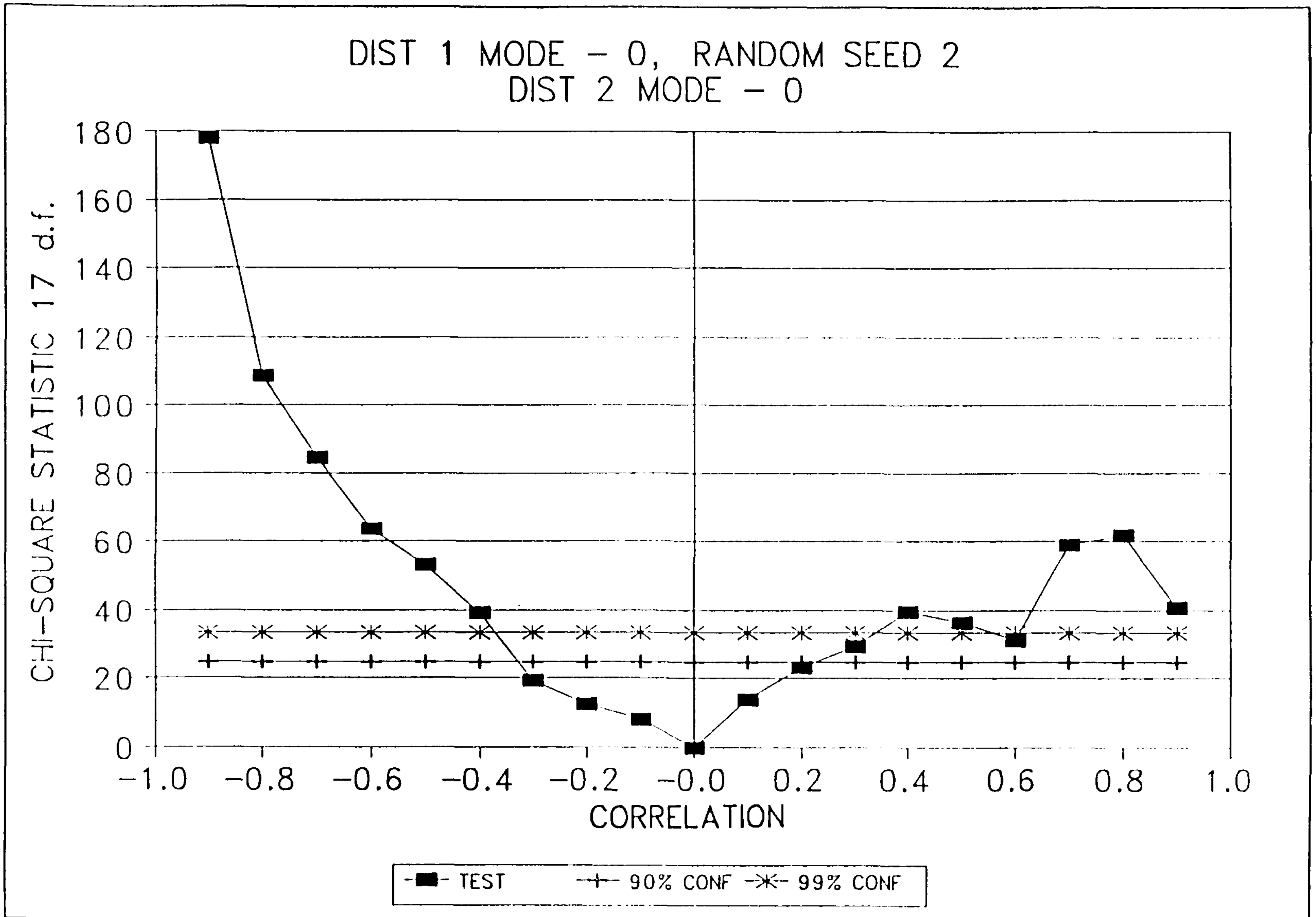


Figure 15 Chi-square test for Case 1 with random seed 2 and 17 d.f.

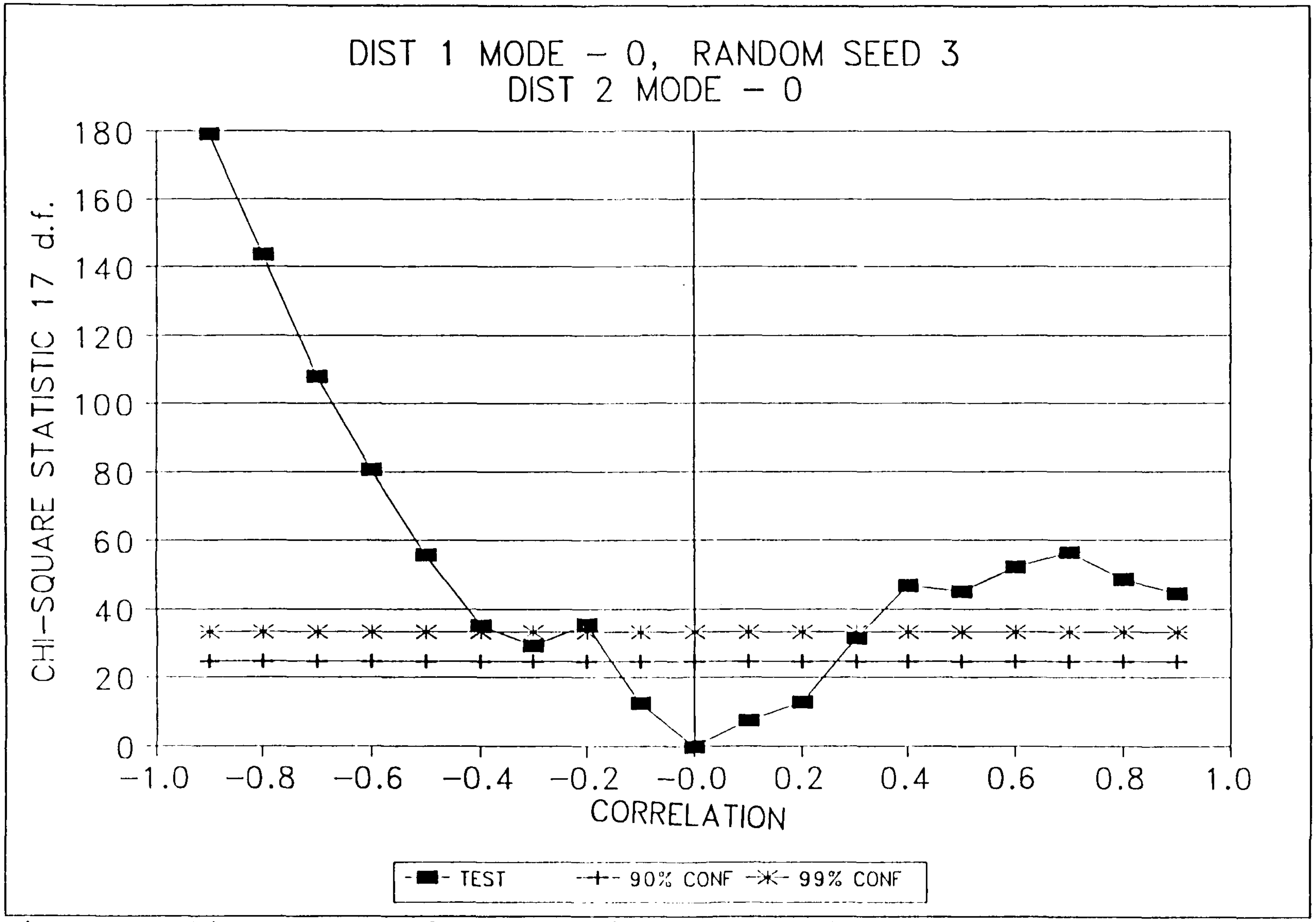


Figure 16 Chi-square test for Case 1 with random seed 3 and 17 d.f.

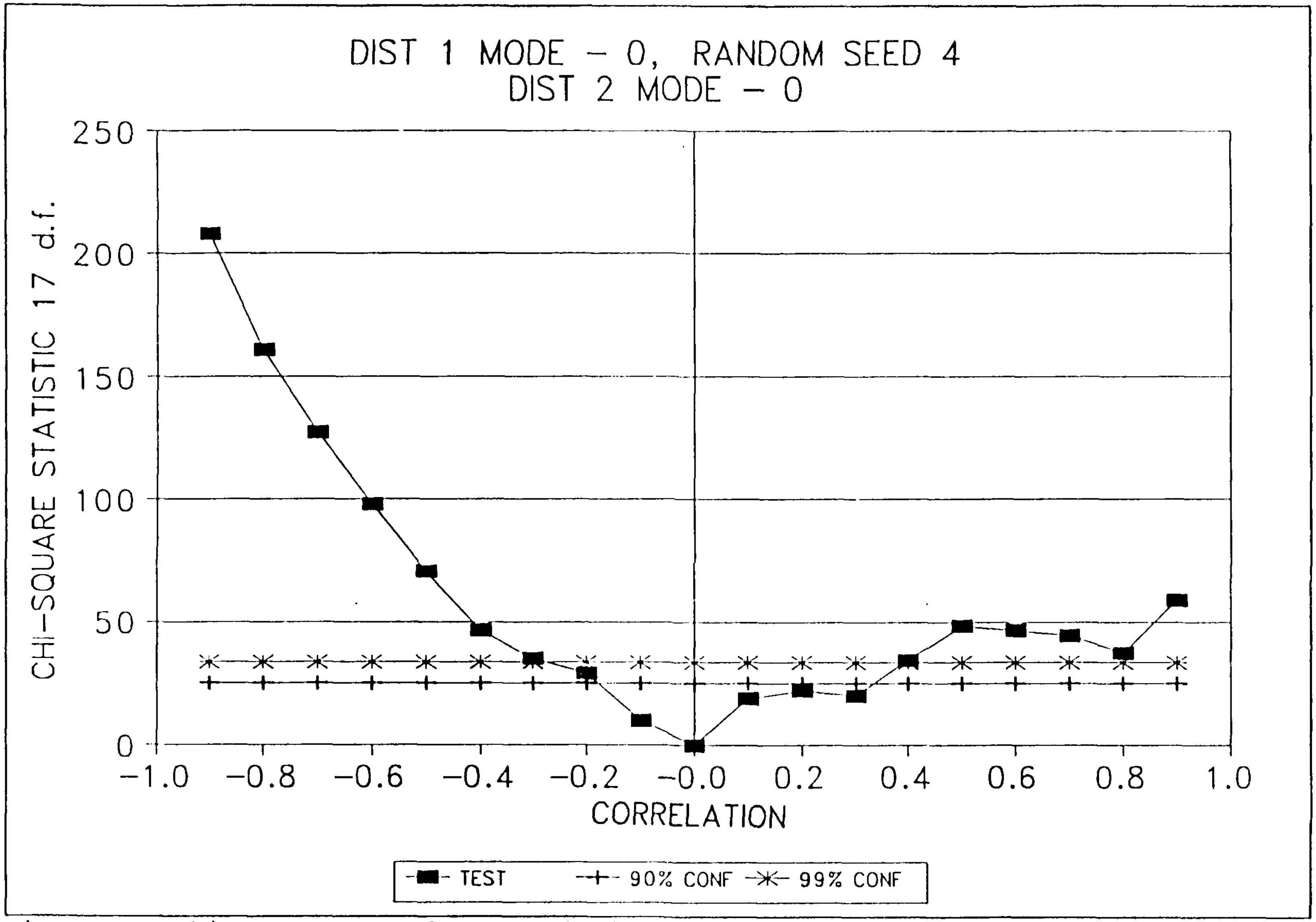


Figure 17 Chi-square test for Case 1 with random seed 4 and 17 d.f.

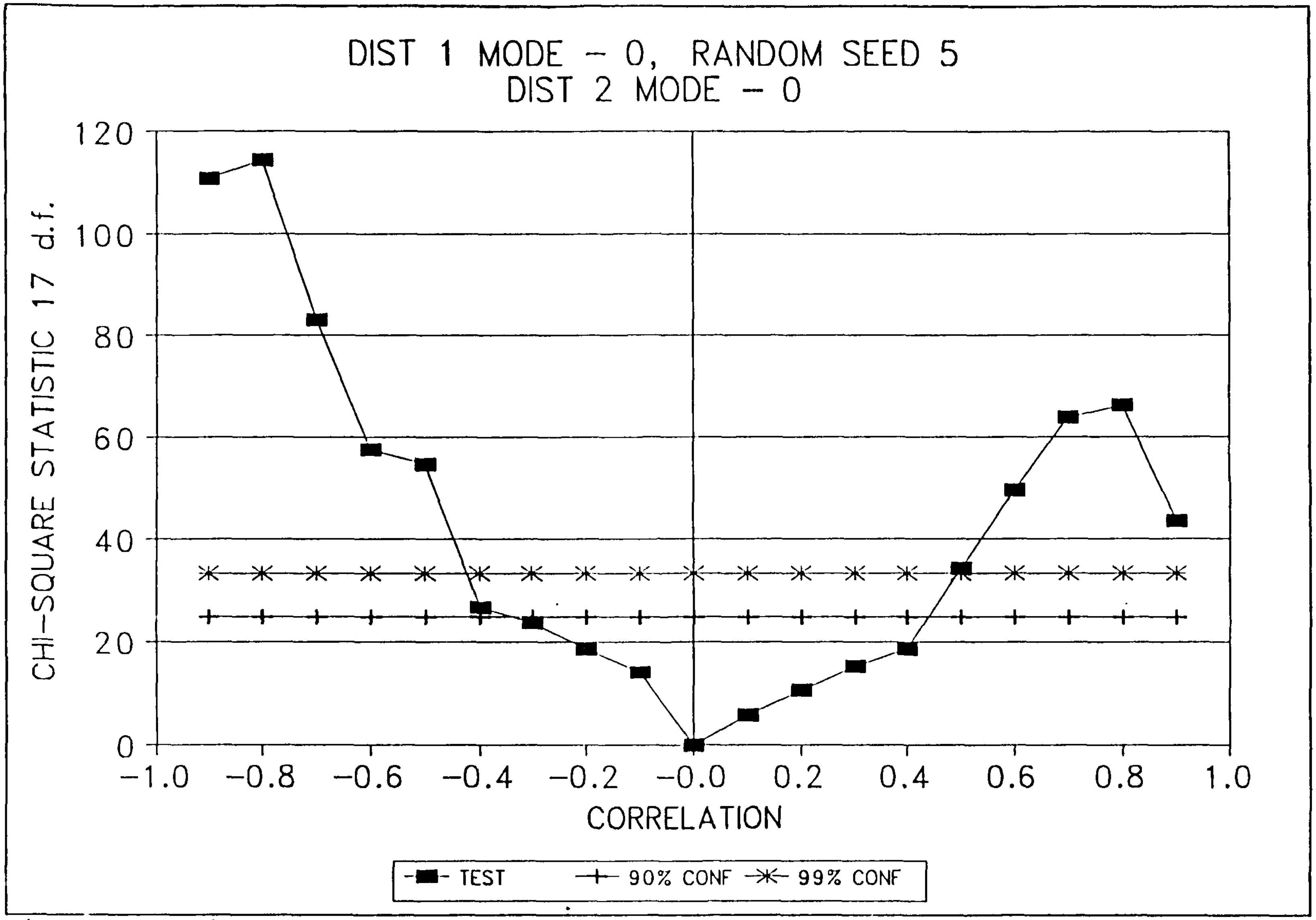


Figure 18 Chi-square test for Case 1 with random seed 5 and 17 d.f.

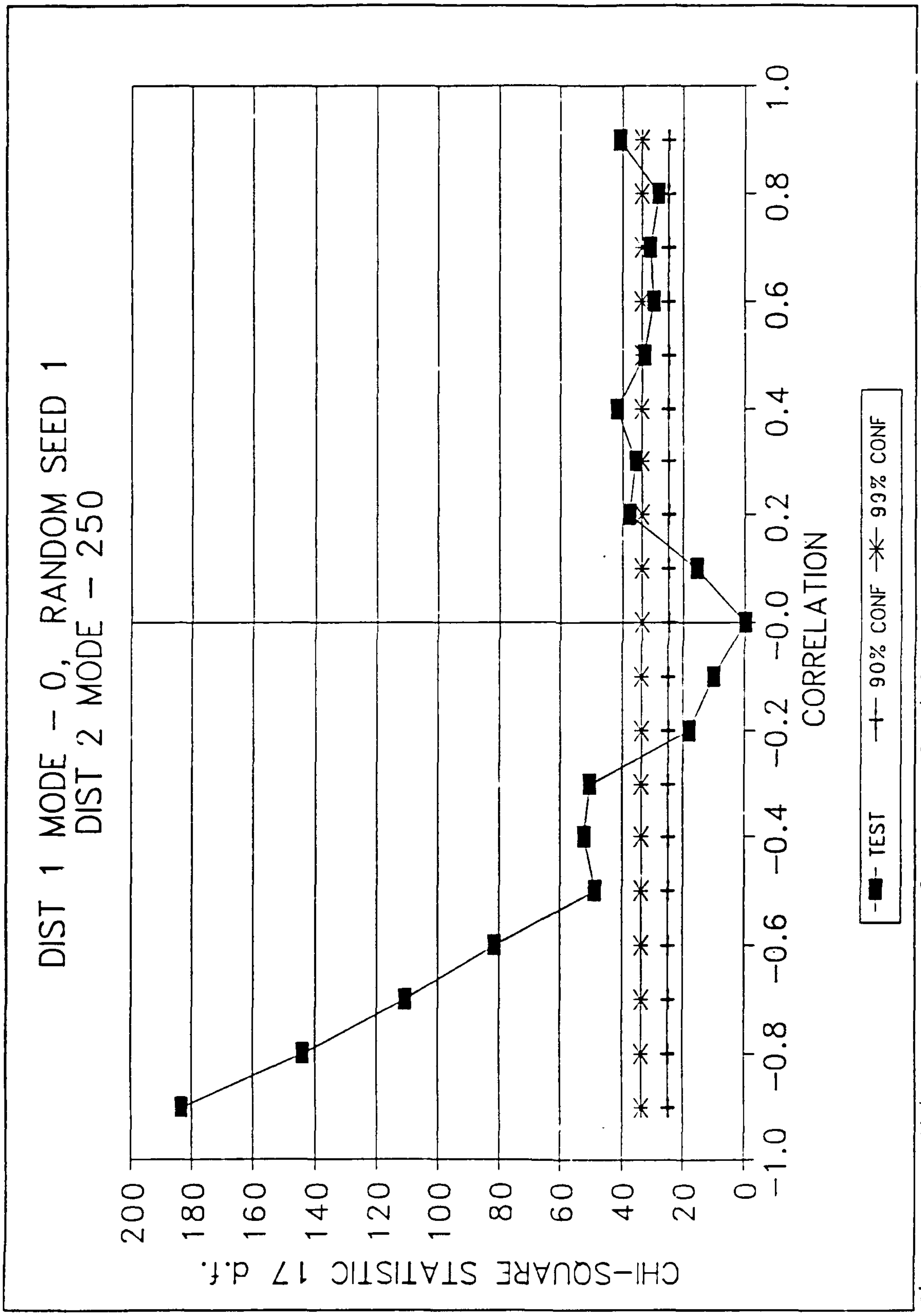


Figure 19 Chi-square test for Case 2 with random seed 1 and 17 d.f.

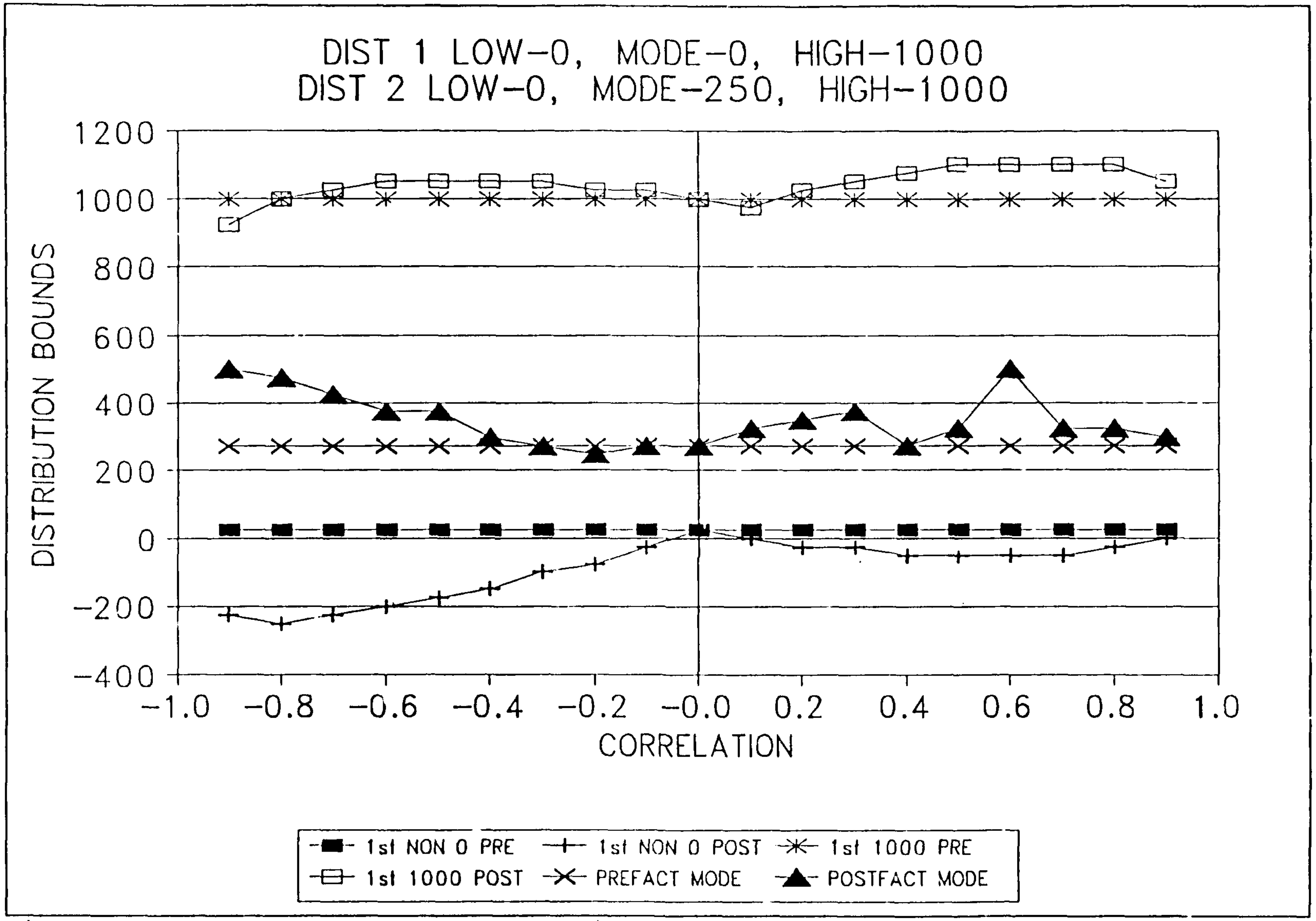


Figure 20 Boundary chart for Case 2 with random seed 1

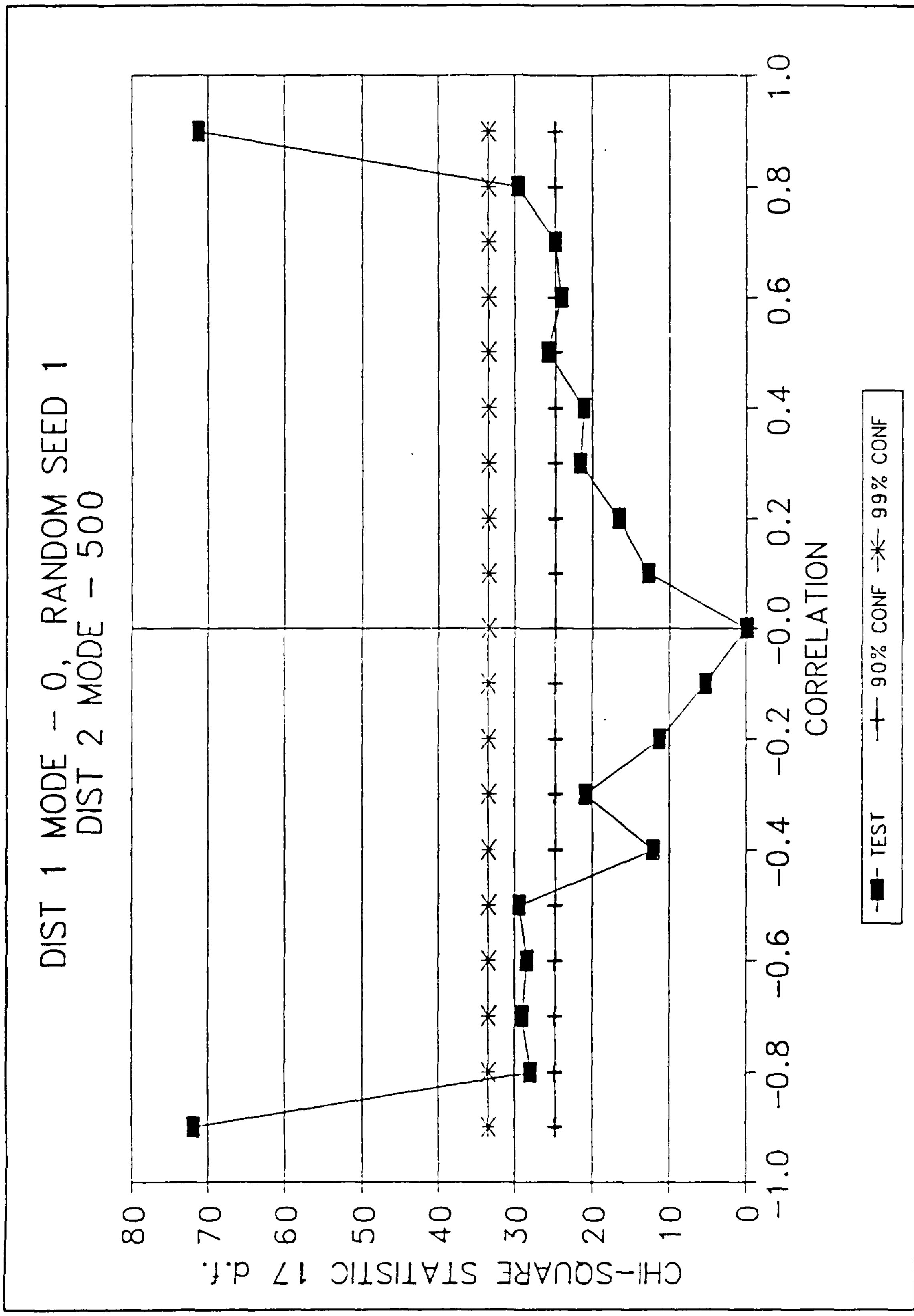


Figure 21 Chi-square test for Case 3 with random seed 1 and 17 d.f.

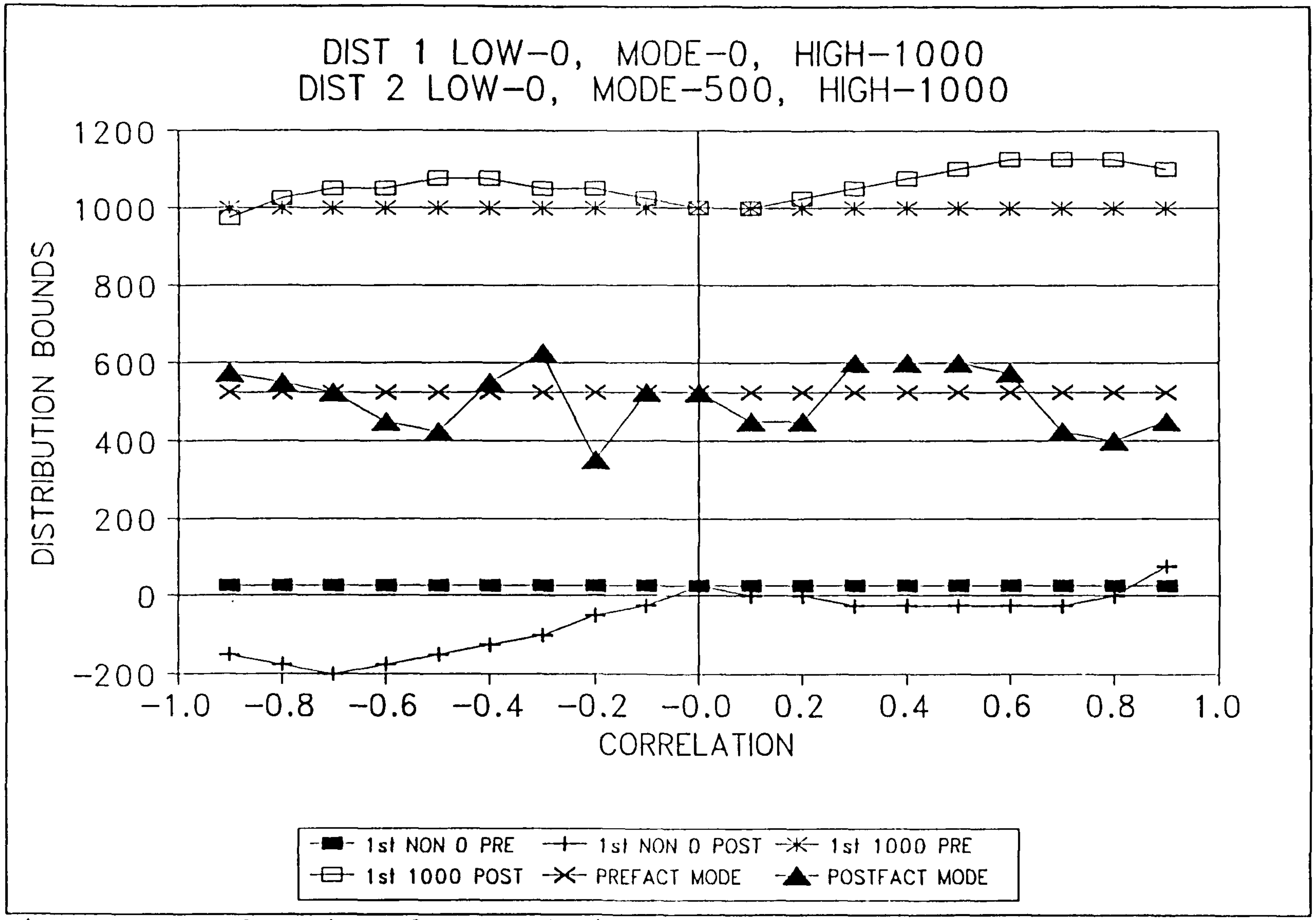


Figure 22 Boundary chart for Case 3 with random seed 1

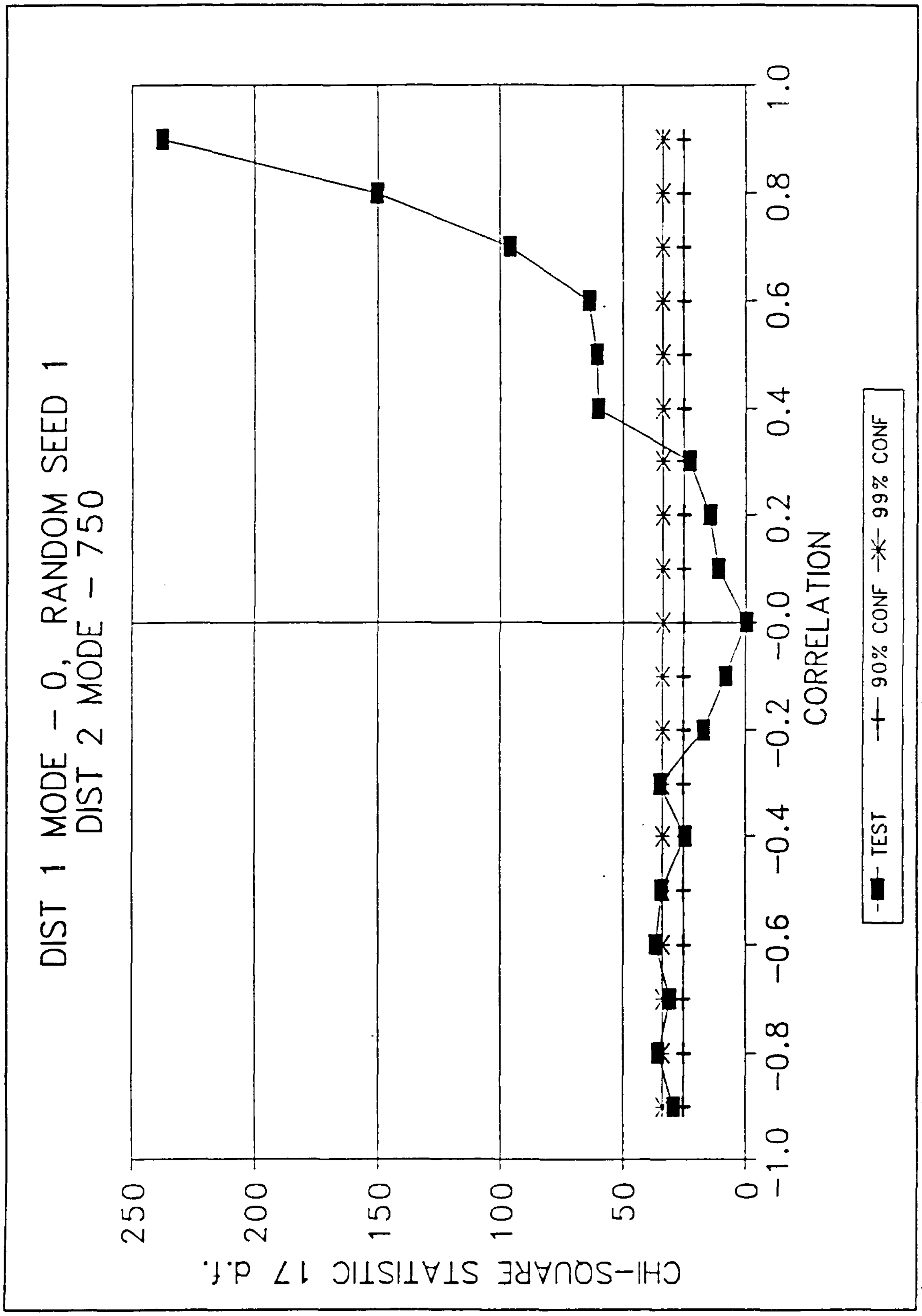


Figure 23 Chi-square test for Case 4 with random seed 1 and 17 d.f.

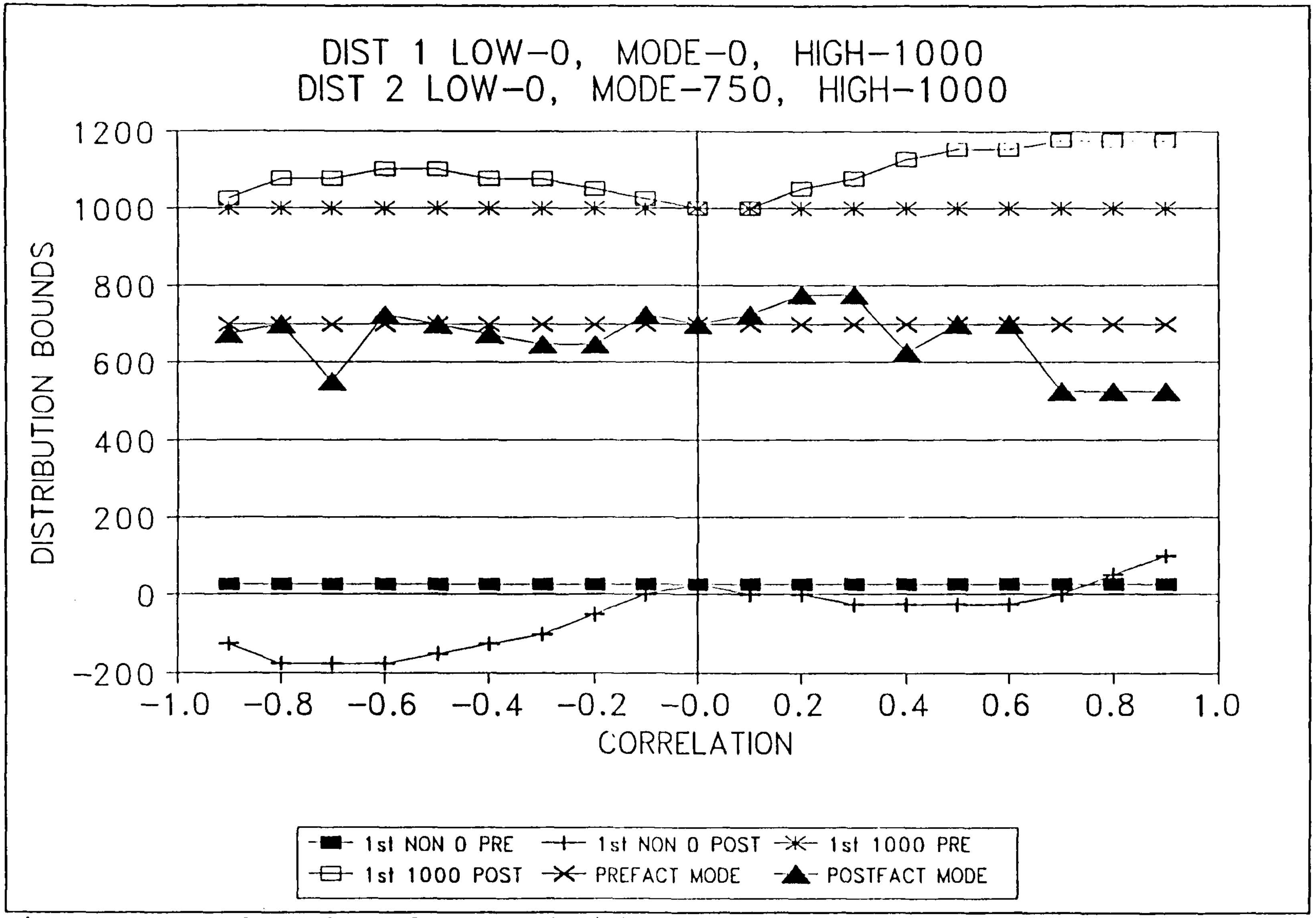


Figure 24 Boundary chart for Case 4 with random seed 1

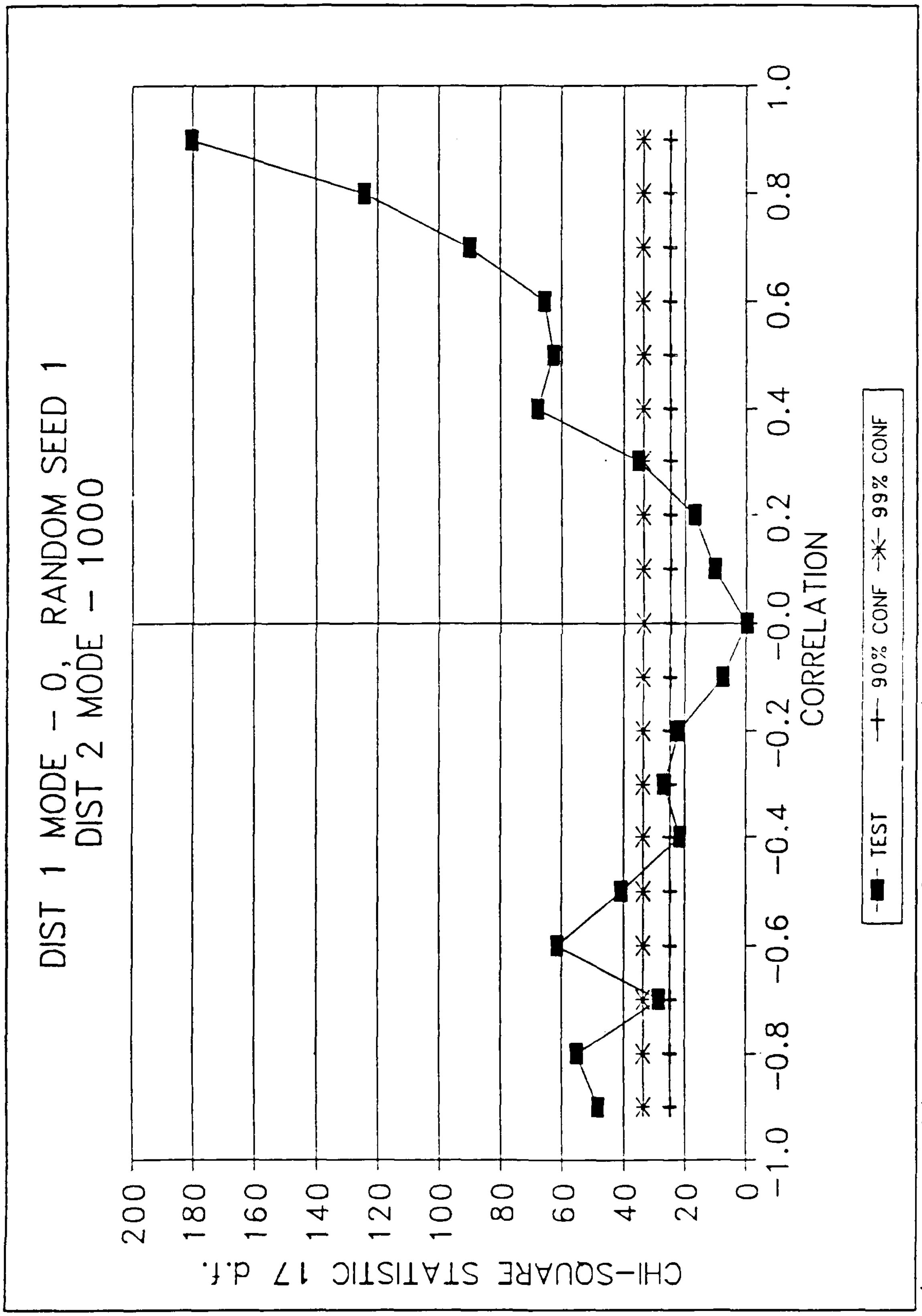


Figure 25 Chi-square test for Case 5 with random seed 1 and 17 d.f.

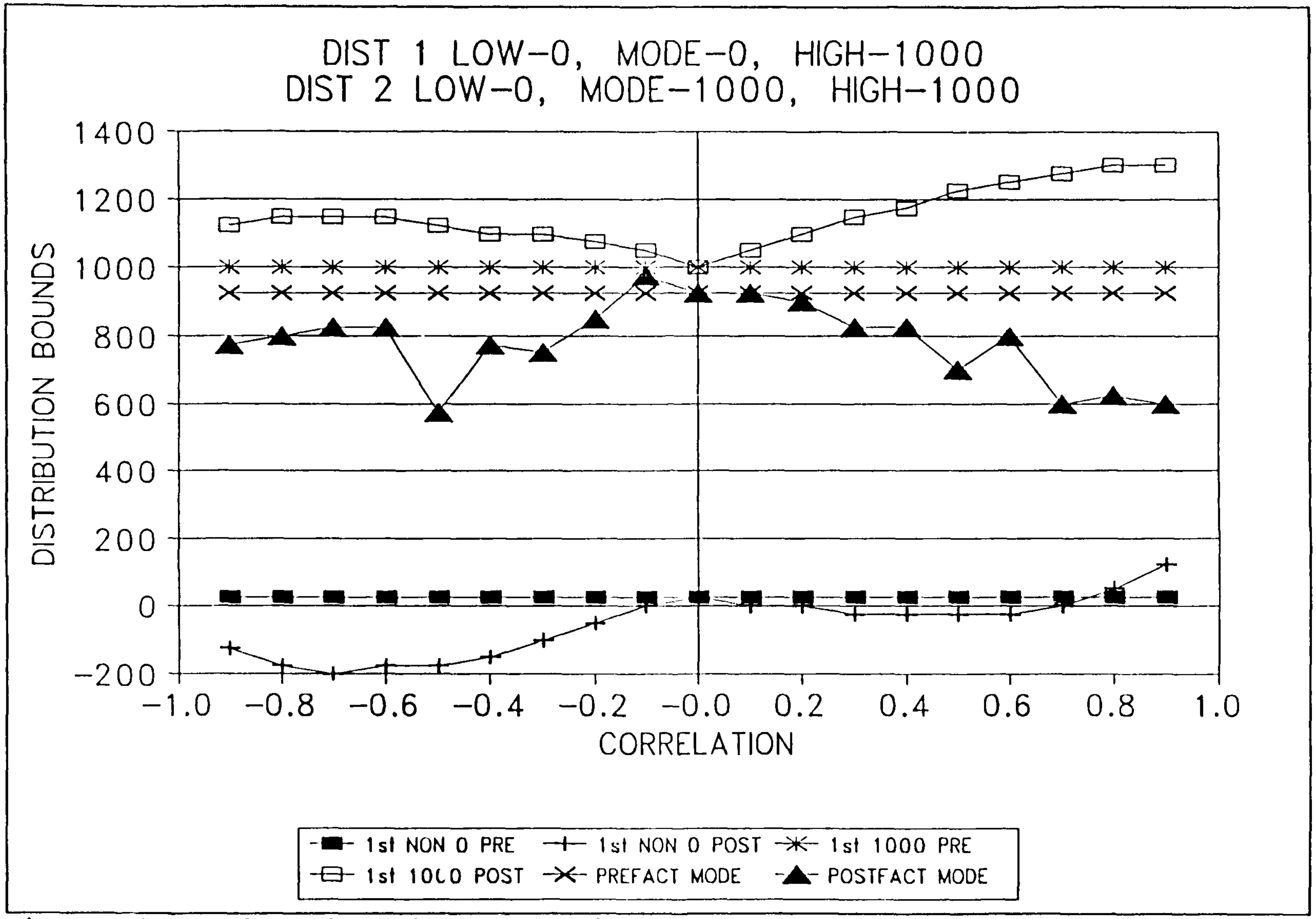


Figure 26 Boundary chart for Case 5 with random seed 1

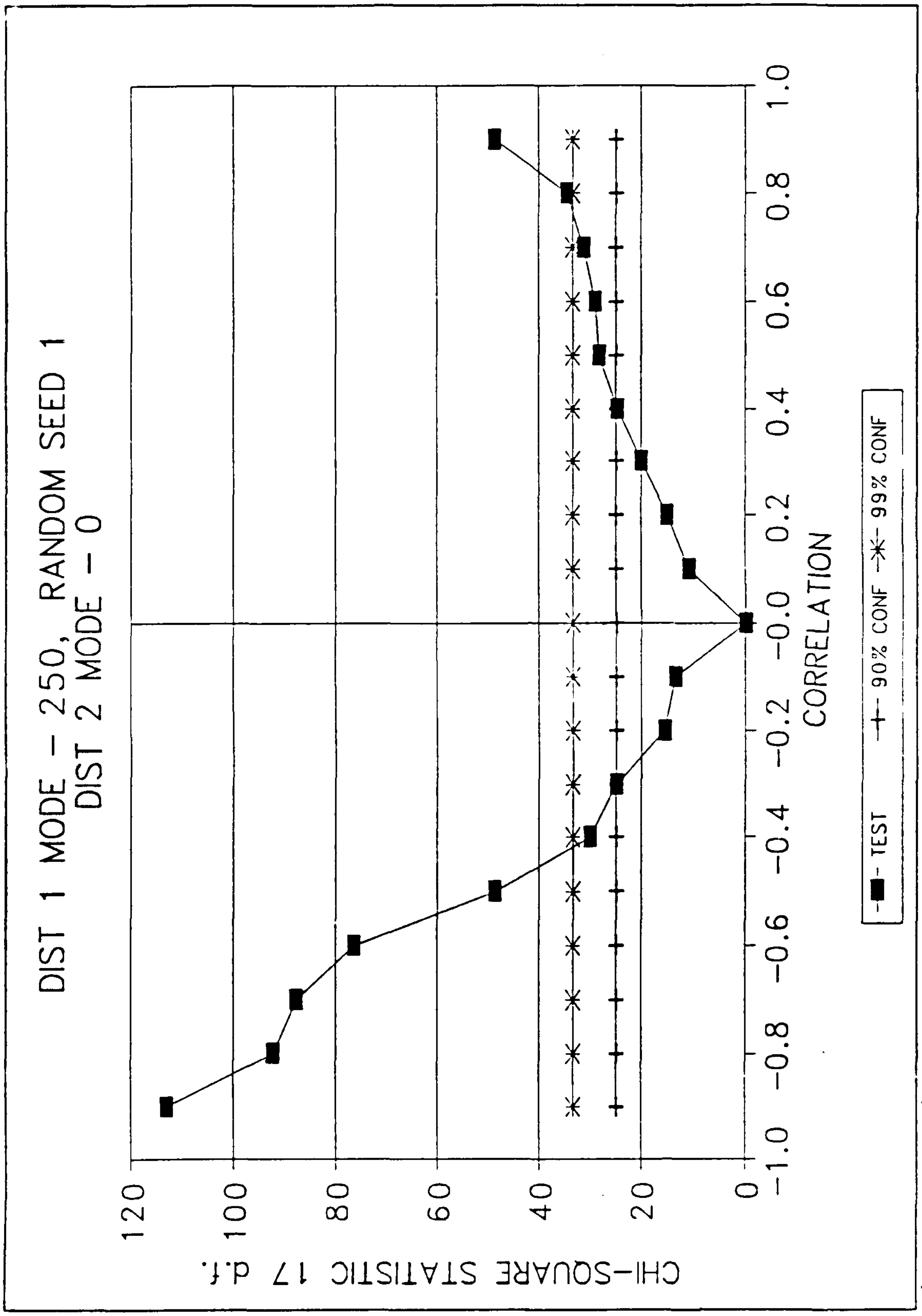


Figure 27 Chi-square test for case 6 with random seed 1 and 17 d.f.

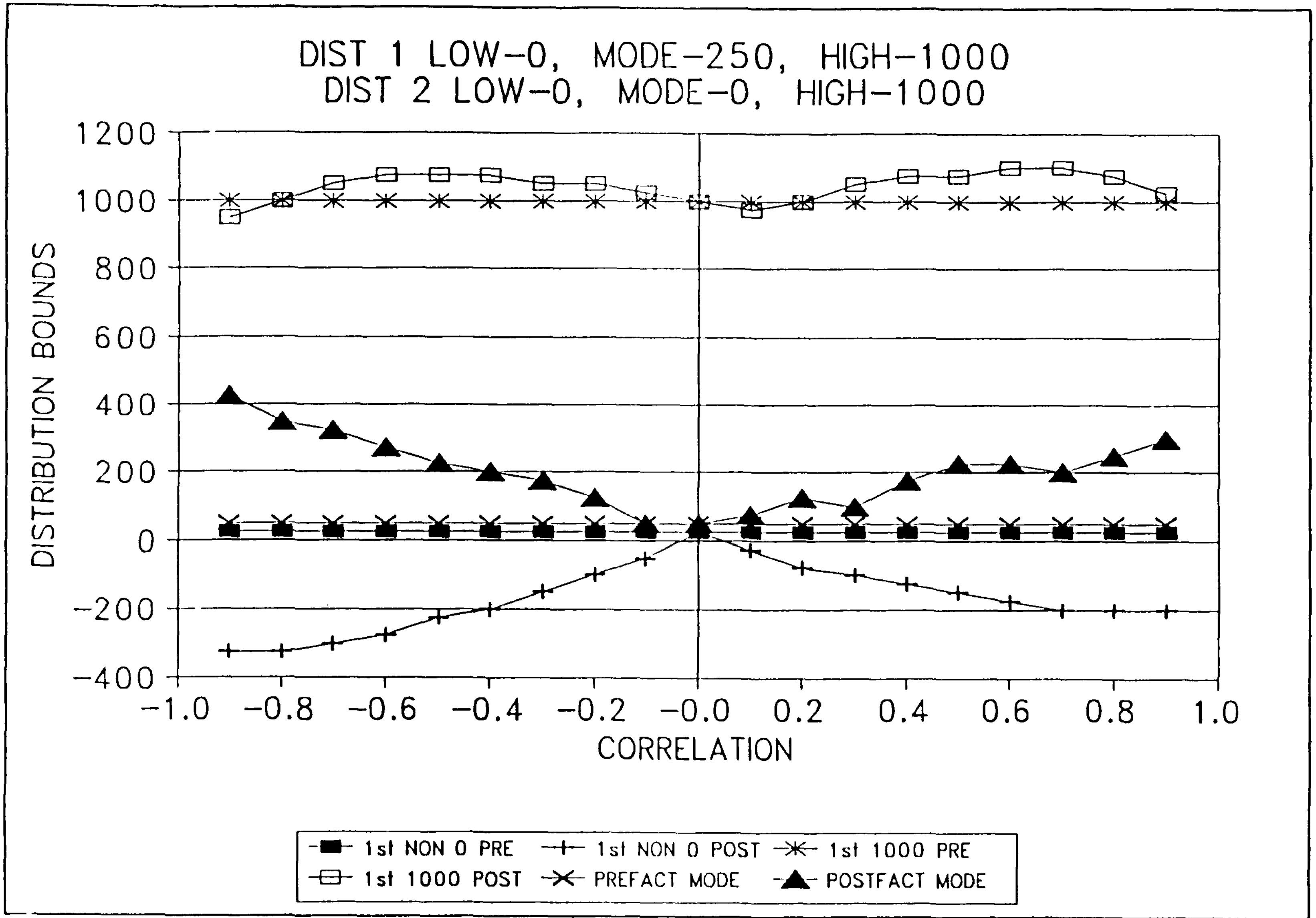


Figure 28 Boundary chart for Case 6 with random seed 1

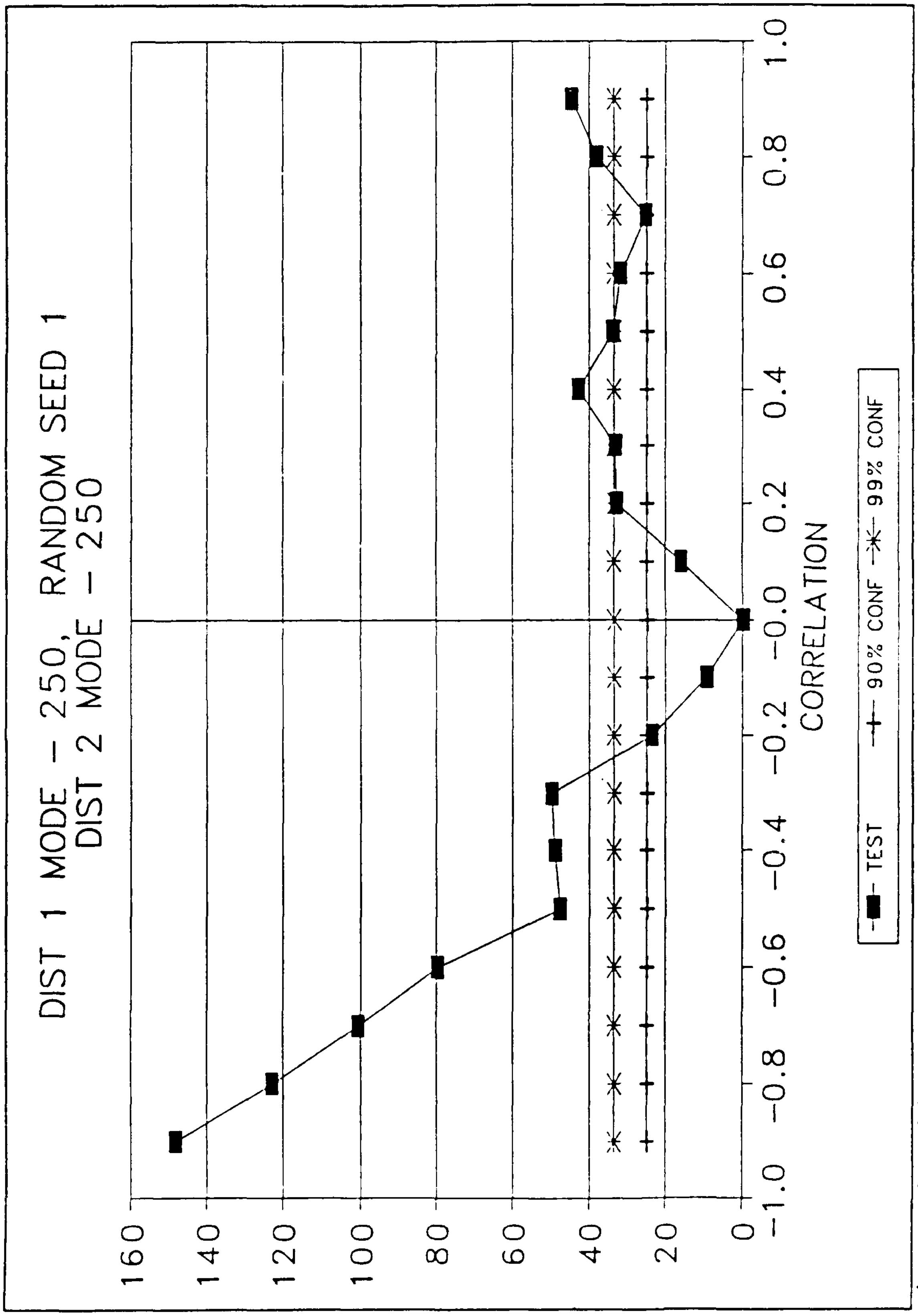


Figure 29 Chi-square test for Case 7 with random seed 1 and 17 d.f.

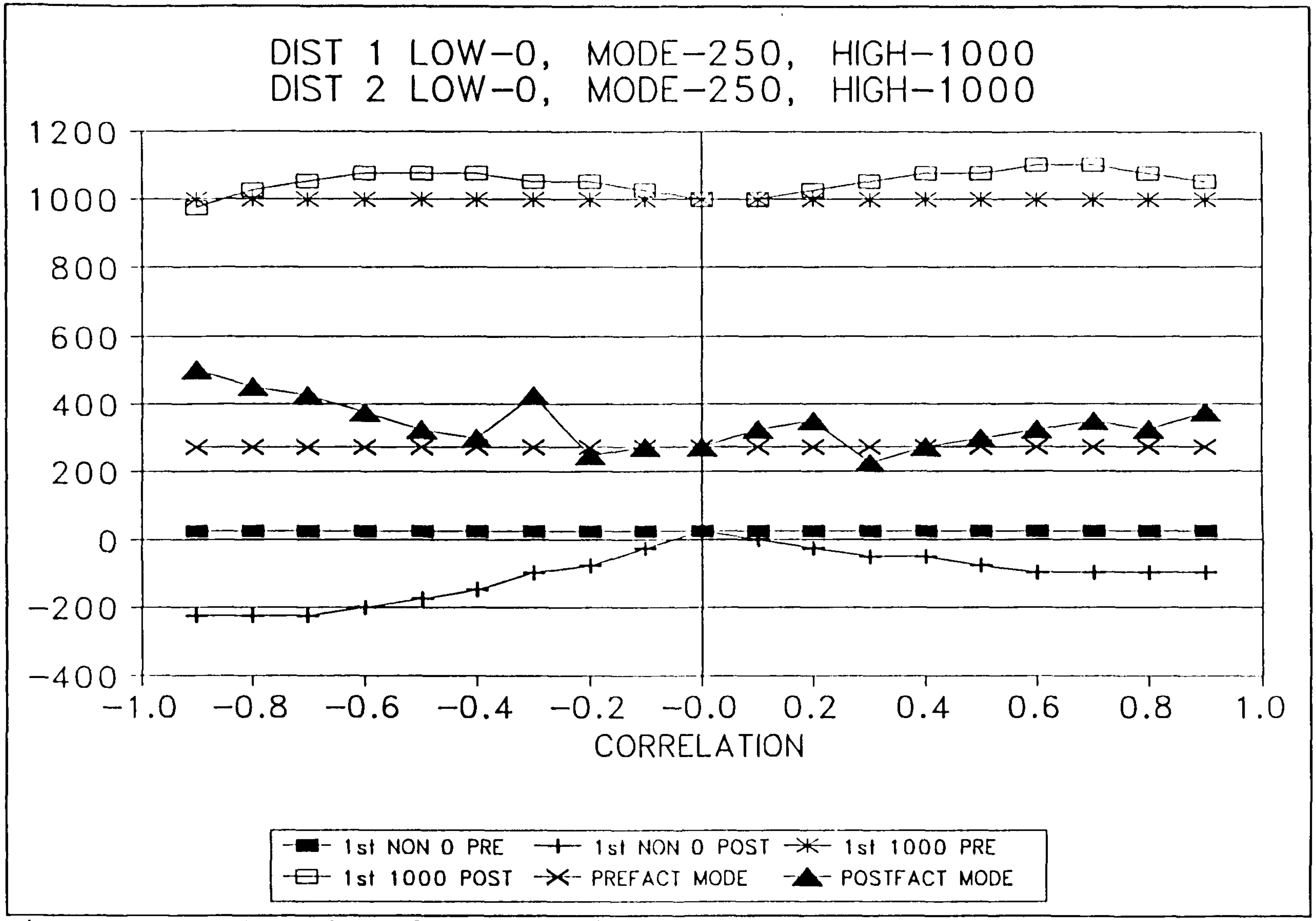


Figure 30 Boundary chart for Case 7 with random seed 1

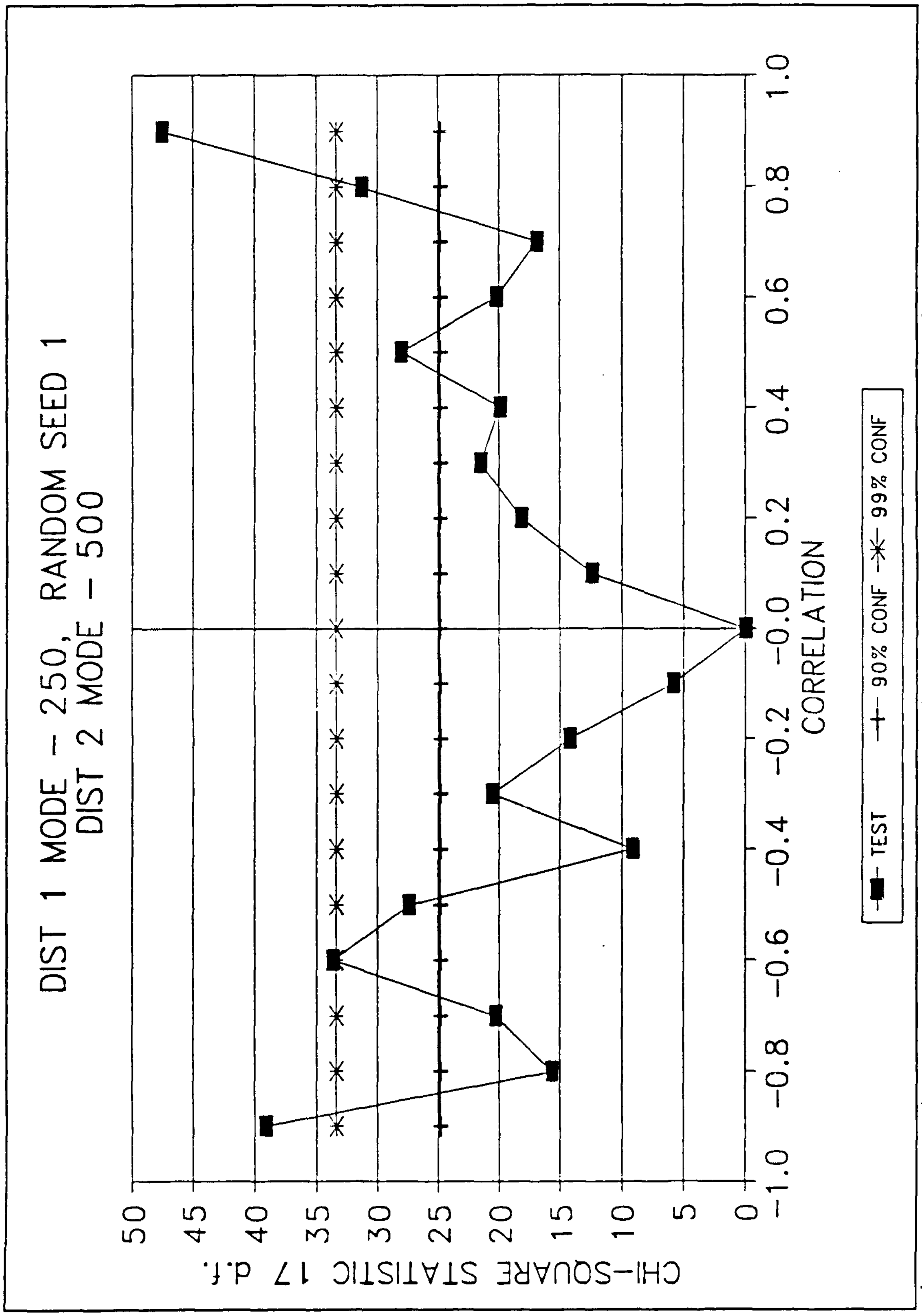


Figure 31 Chi-square test for Case 8 with random seed 1 and 17 d.f.

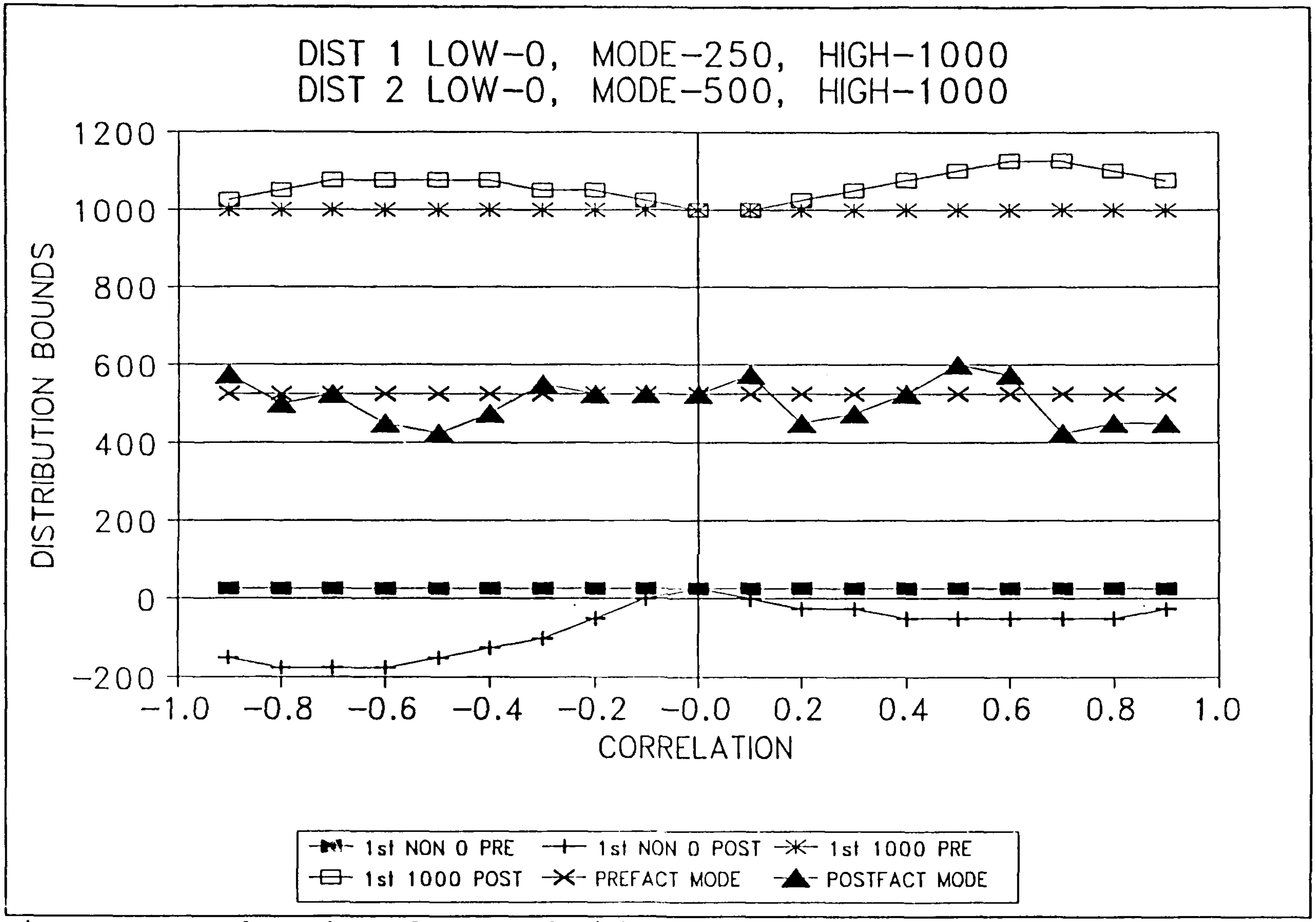


Figure 32 Boundary chart for Case 8 with random seed 1

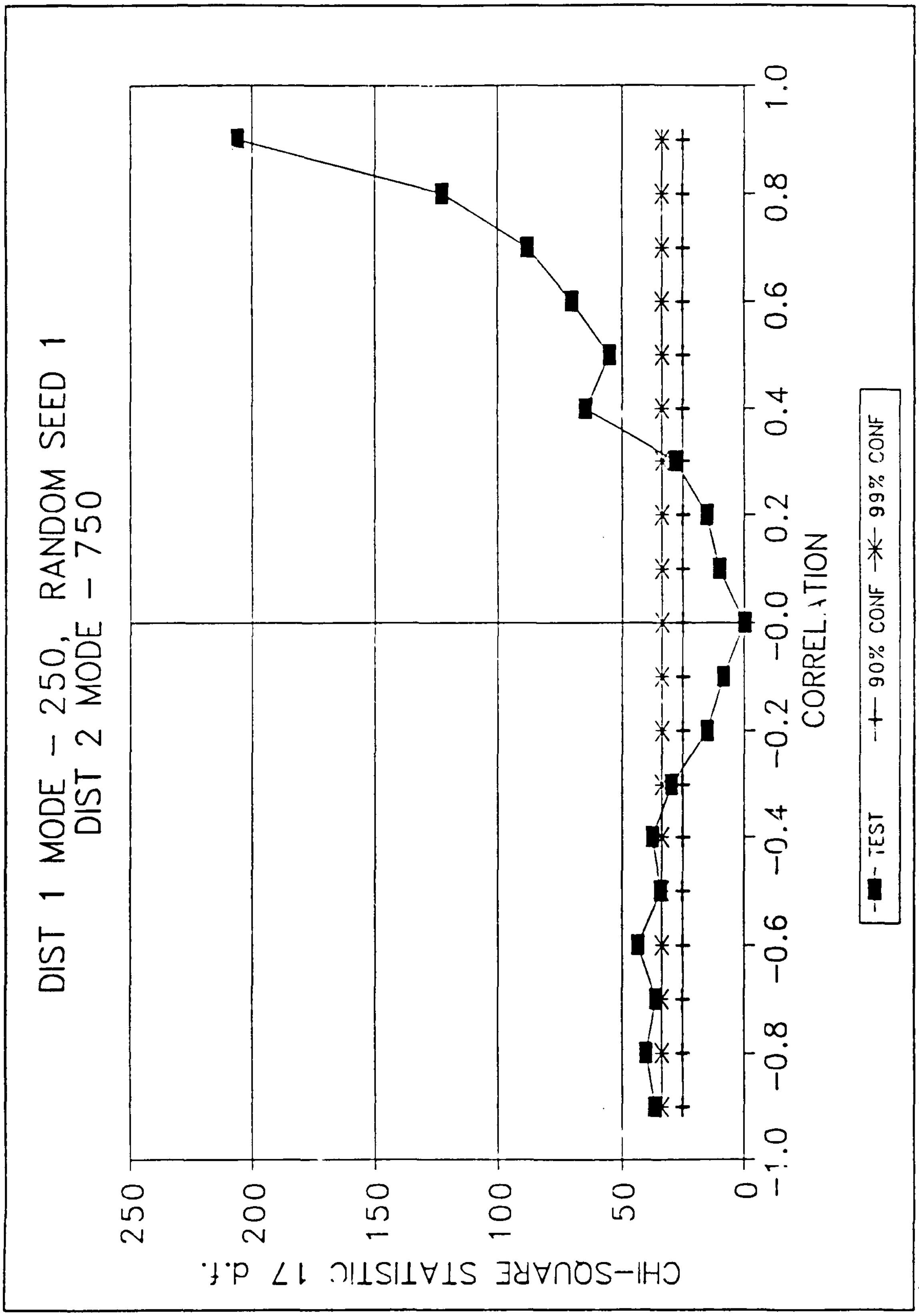


Figure 33 Chi-square test for Case 9 with random seed 1 and 17 d.f.

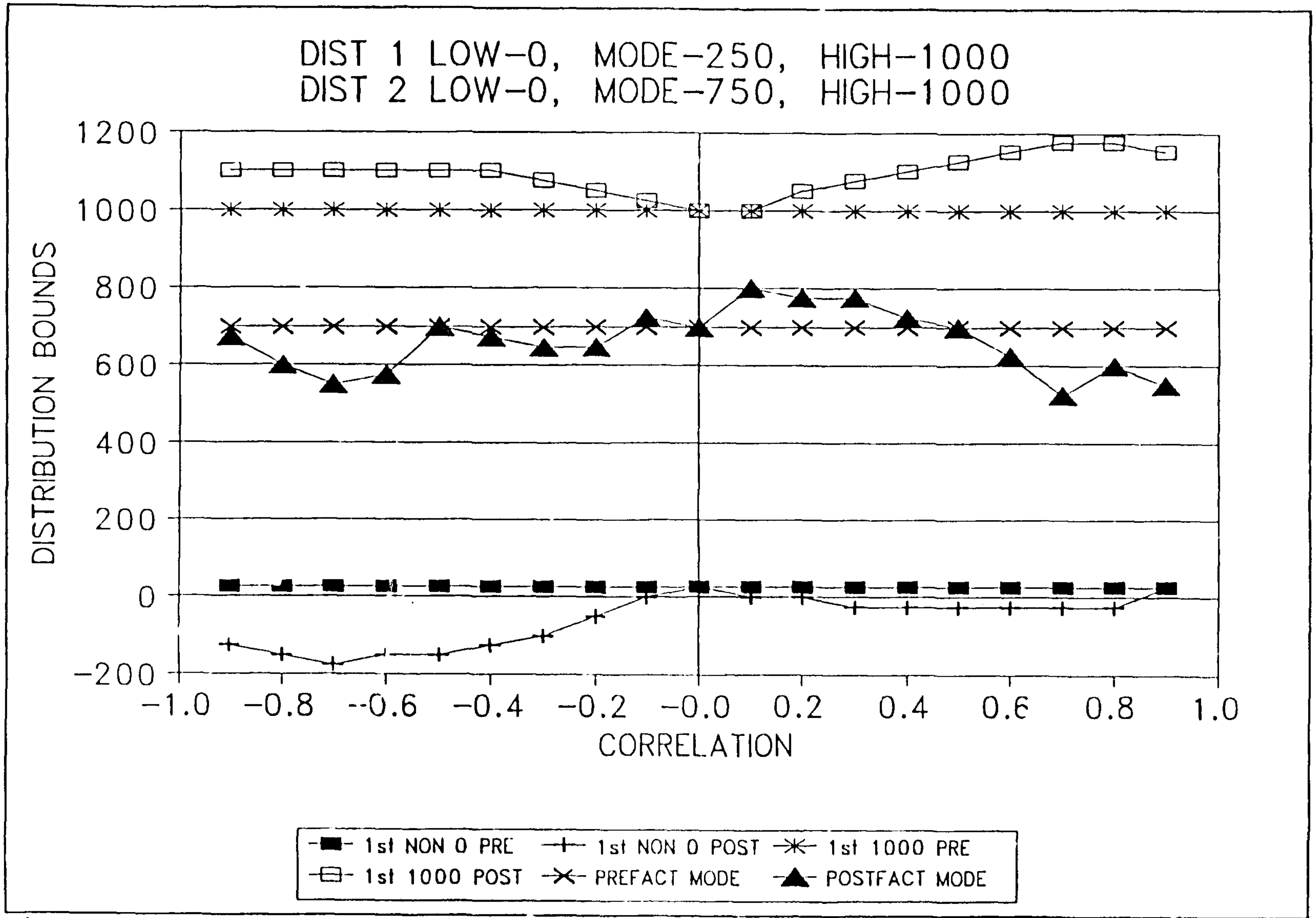


Figure 34 Boundary chart for Case 9 with random seed 1

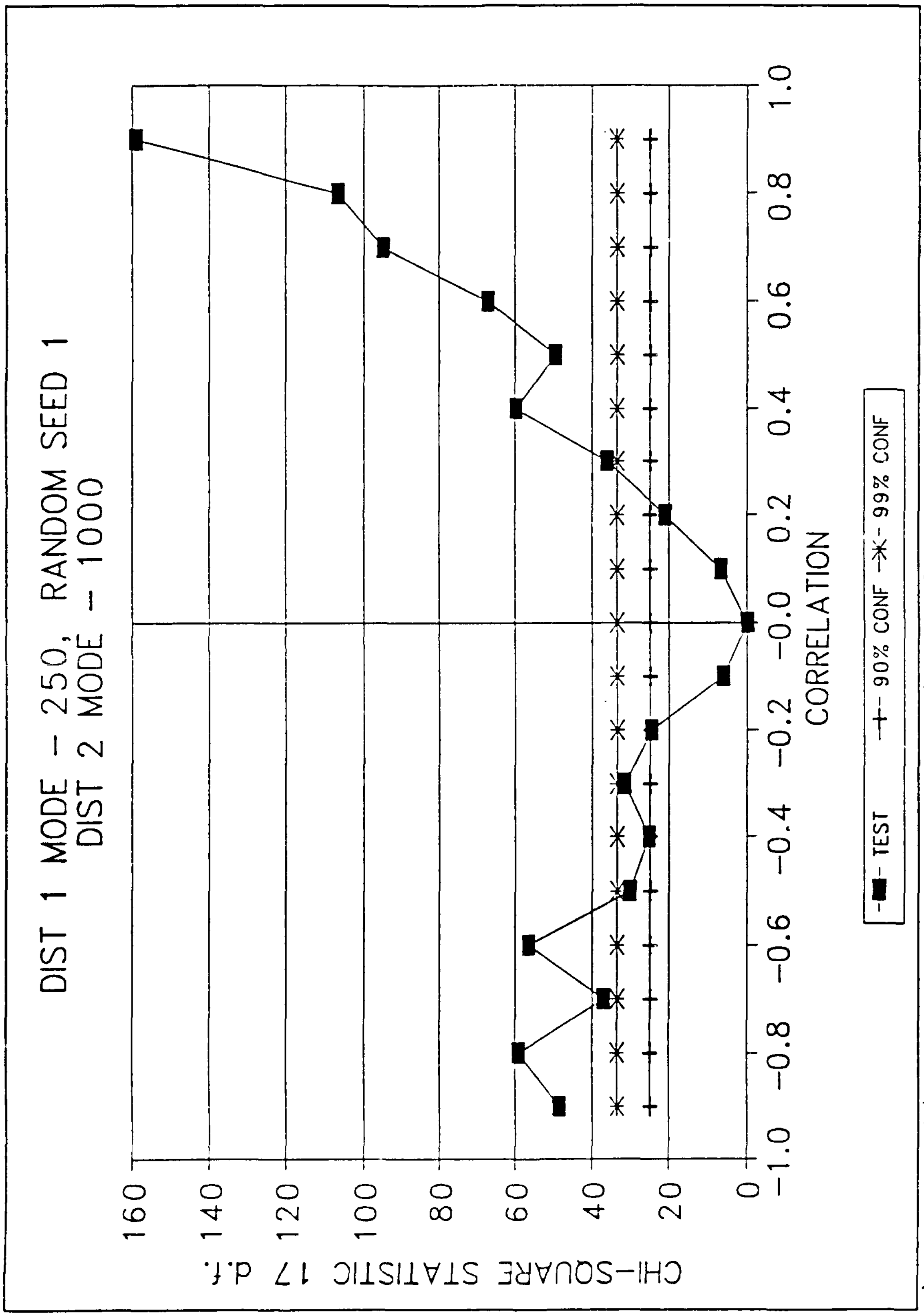


Figure 35 Chi-square test for Case 10 with random seed 1 and 17 d.f.

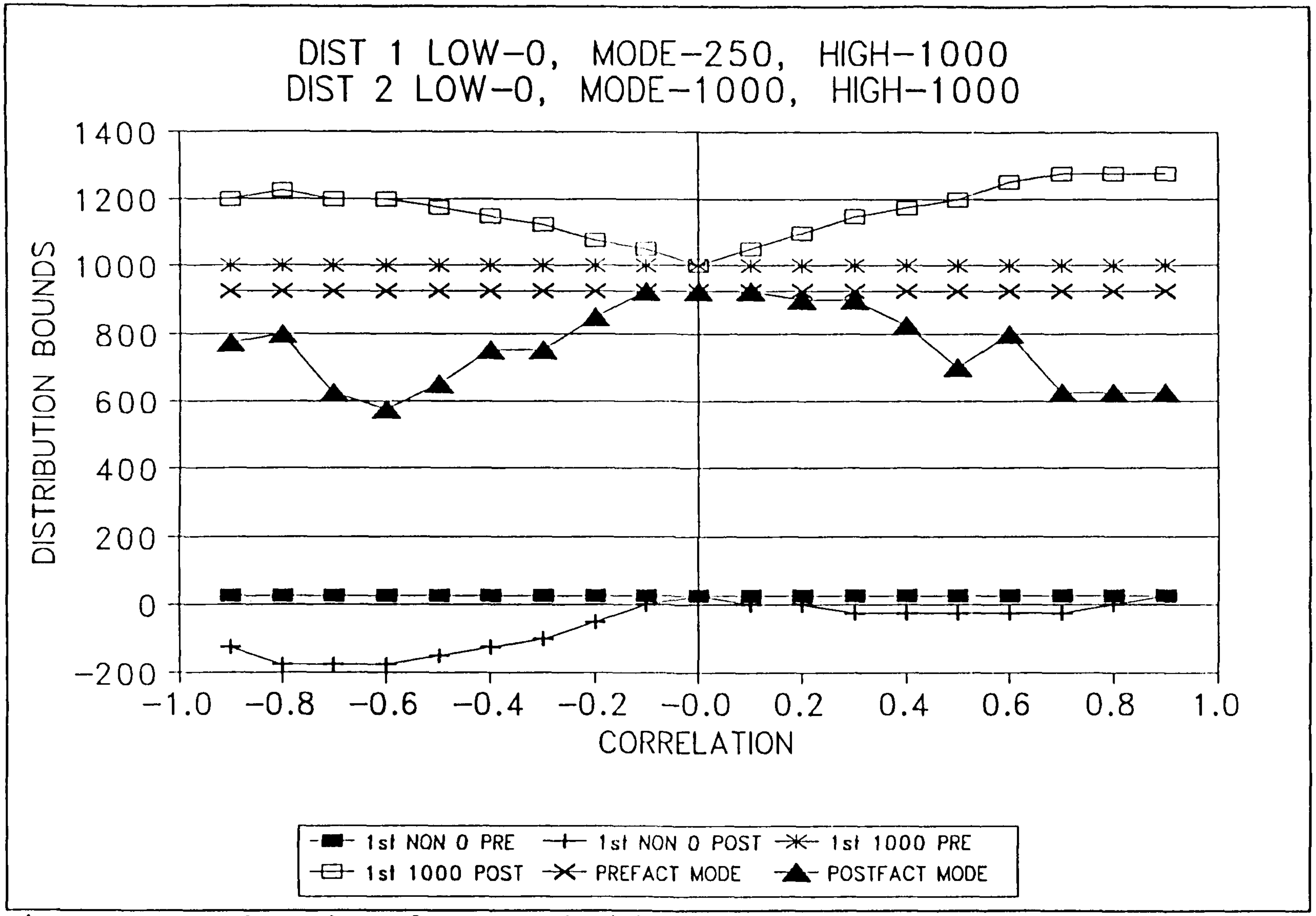


Figure 36 Boundary chart for Case 10 with random seed 1

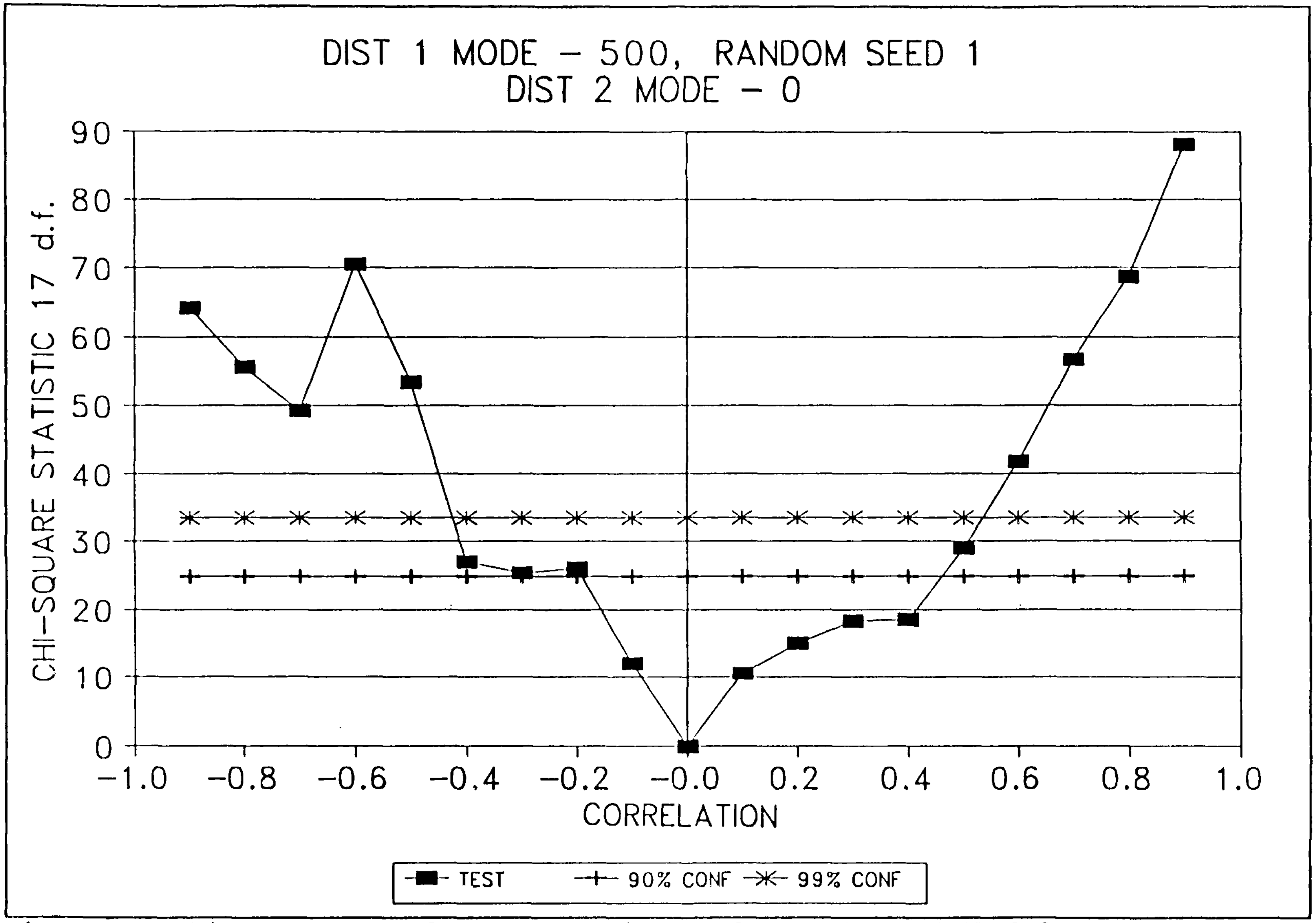


Figure 37 Chi-square test for Case 11 with random seed 1 and 17 d.f.

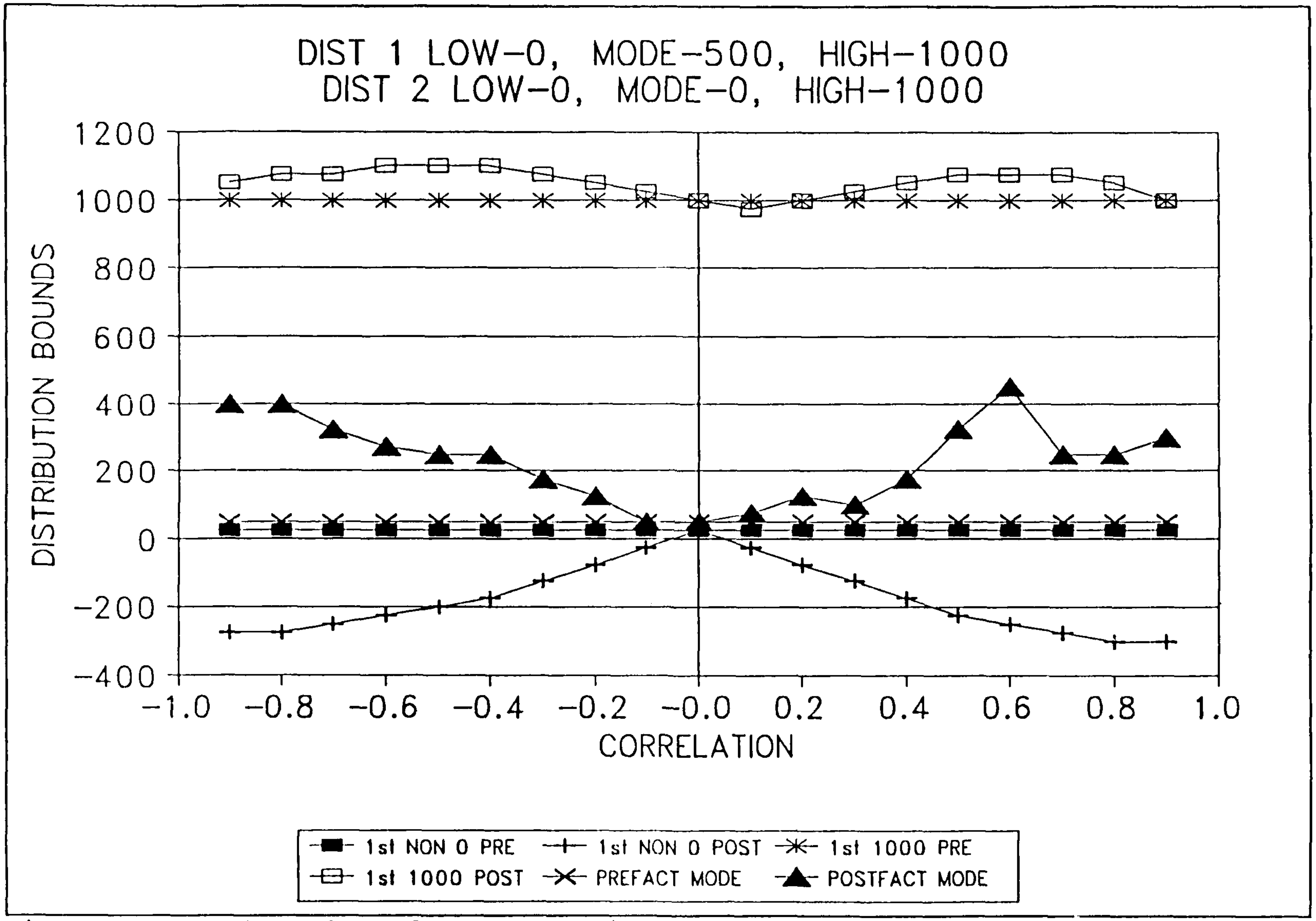


Figure 38 Boundary chart for Case 11 with random seed 1

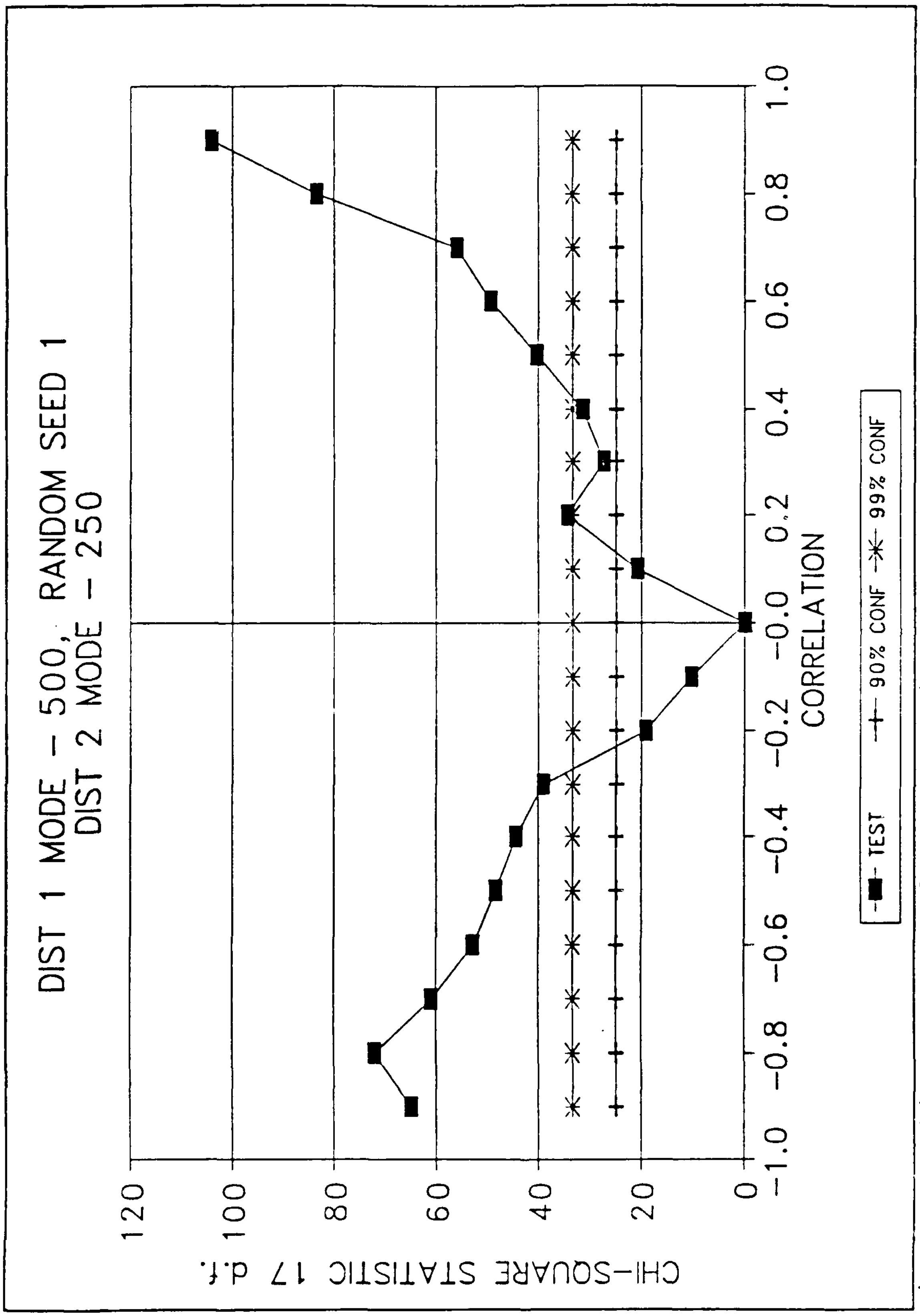


Figure 39 Chi-square test for Case 12 with random seed 1 and 17 d.f.

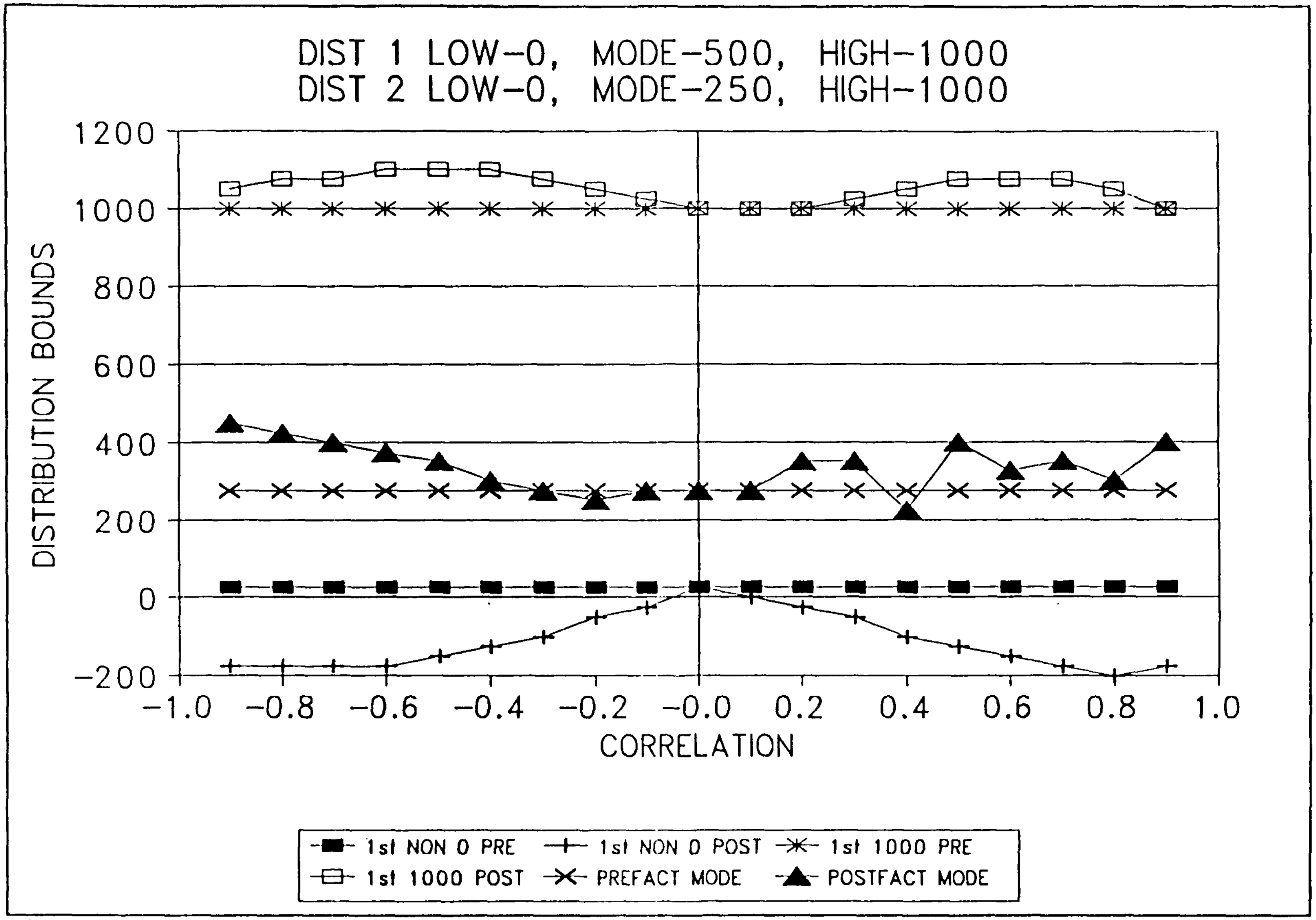


Figure 40 Boundary chart for Case 12 with random seed 1

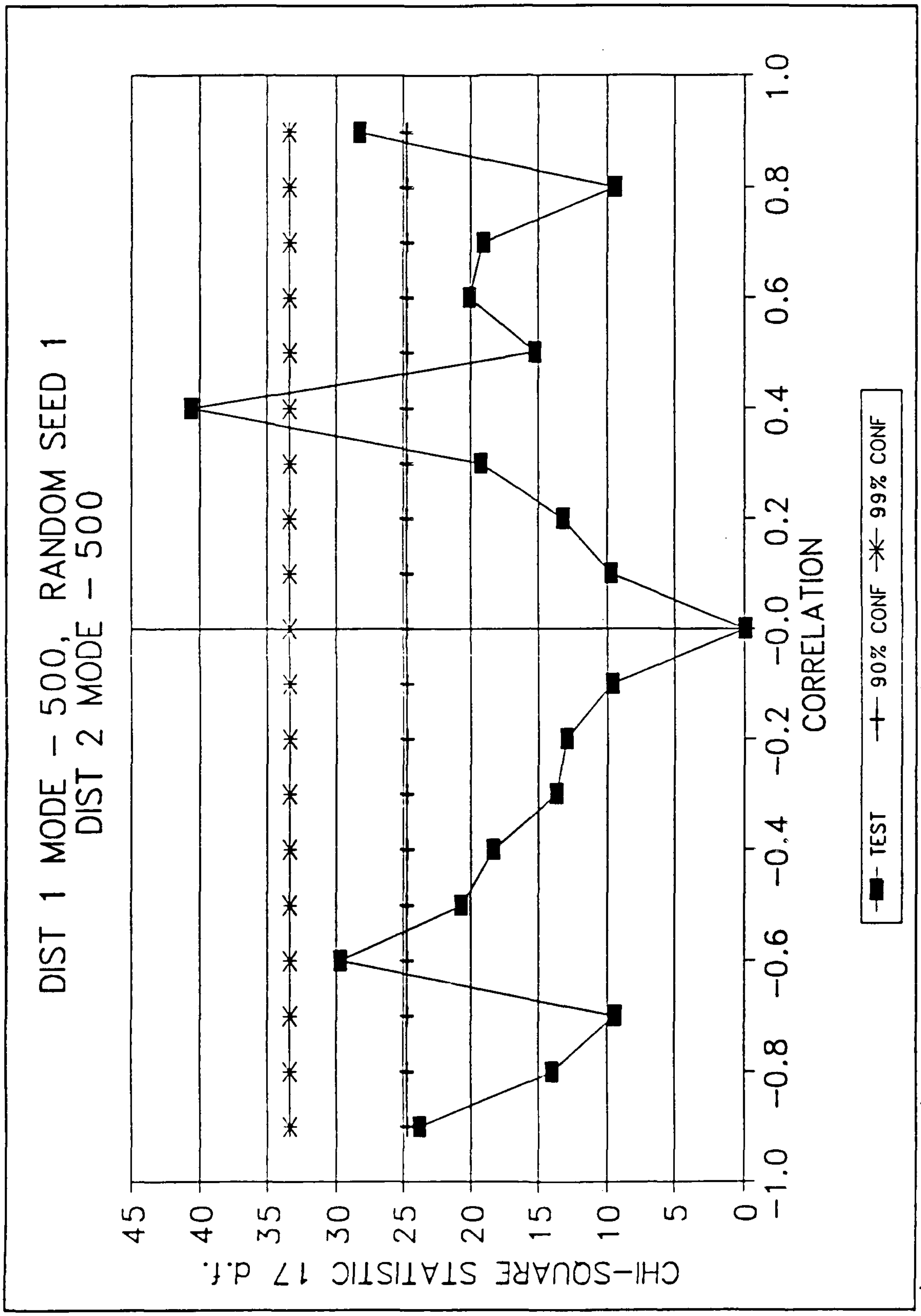


Figure 41 Chi-square test for Case 13 with random seed 1 and 17 d.f.

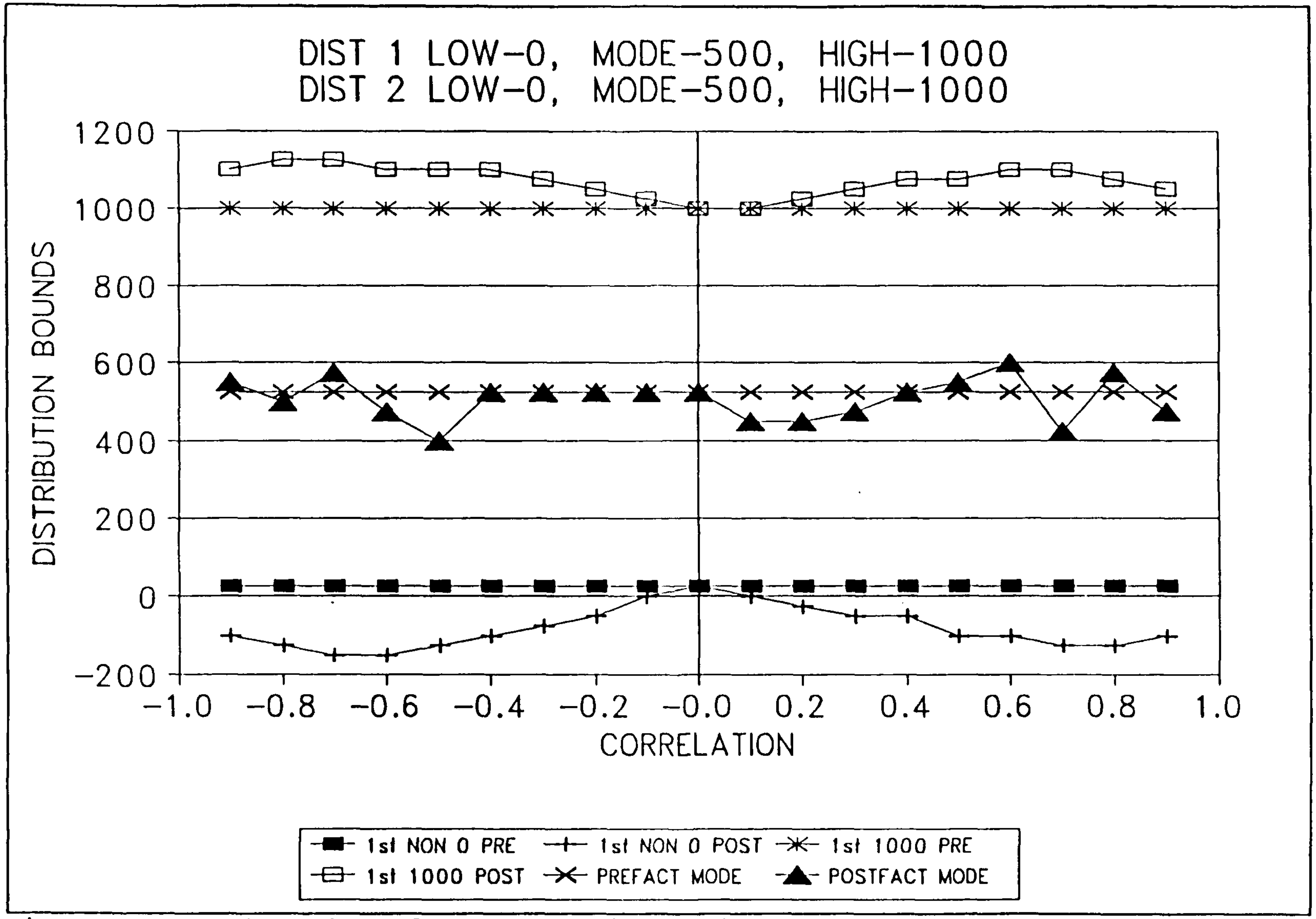


Figure 42 Boundary chart for Case 13 with random seed 1

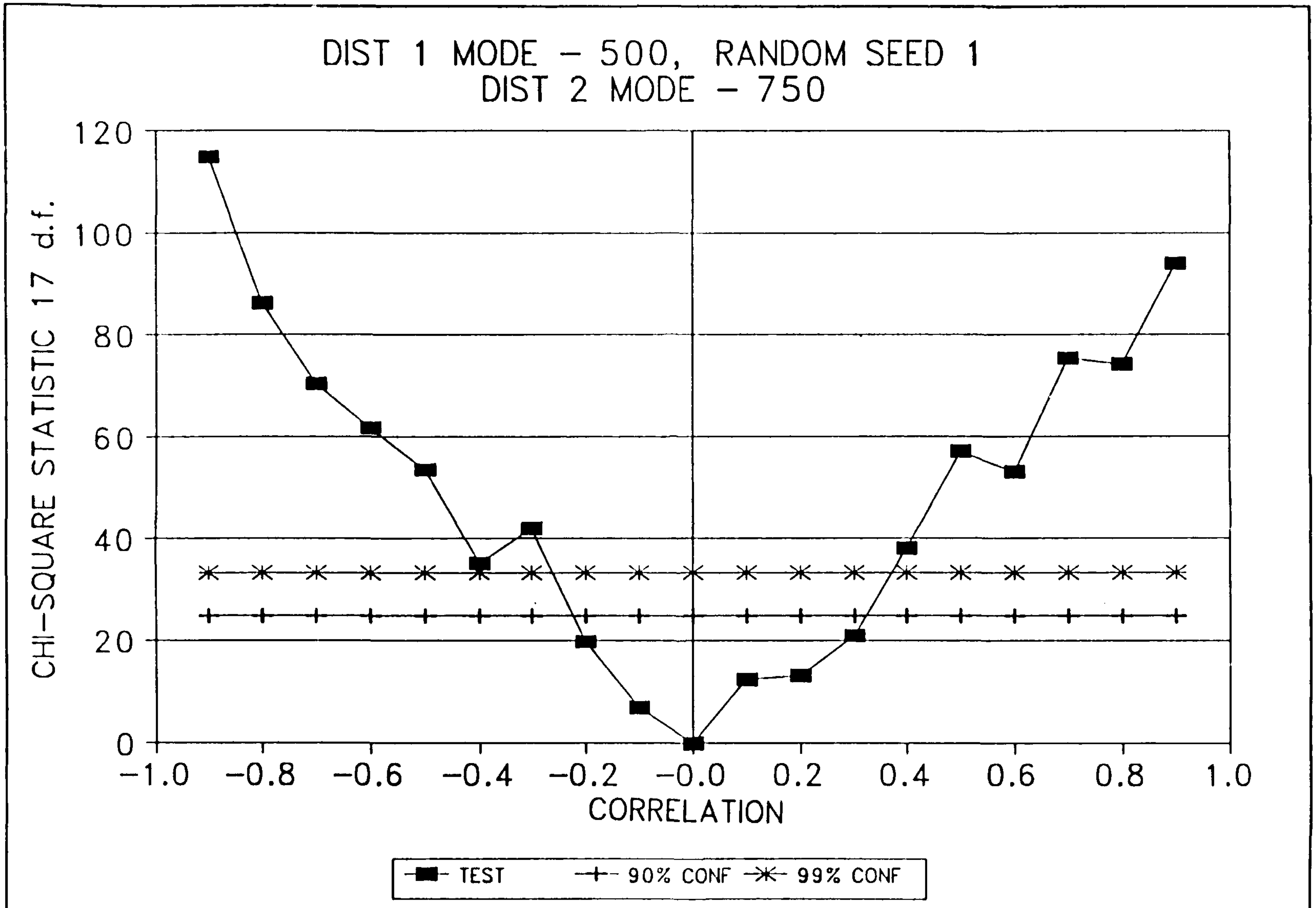


Figure 43 Chi-square test for Case 14 with random seed 1 and 17 d.f.

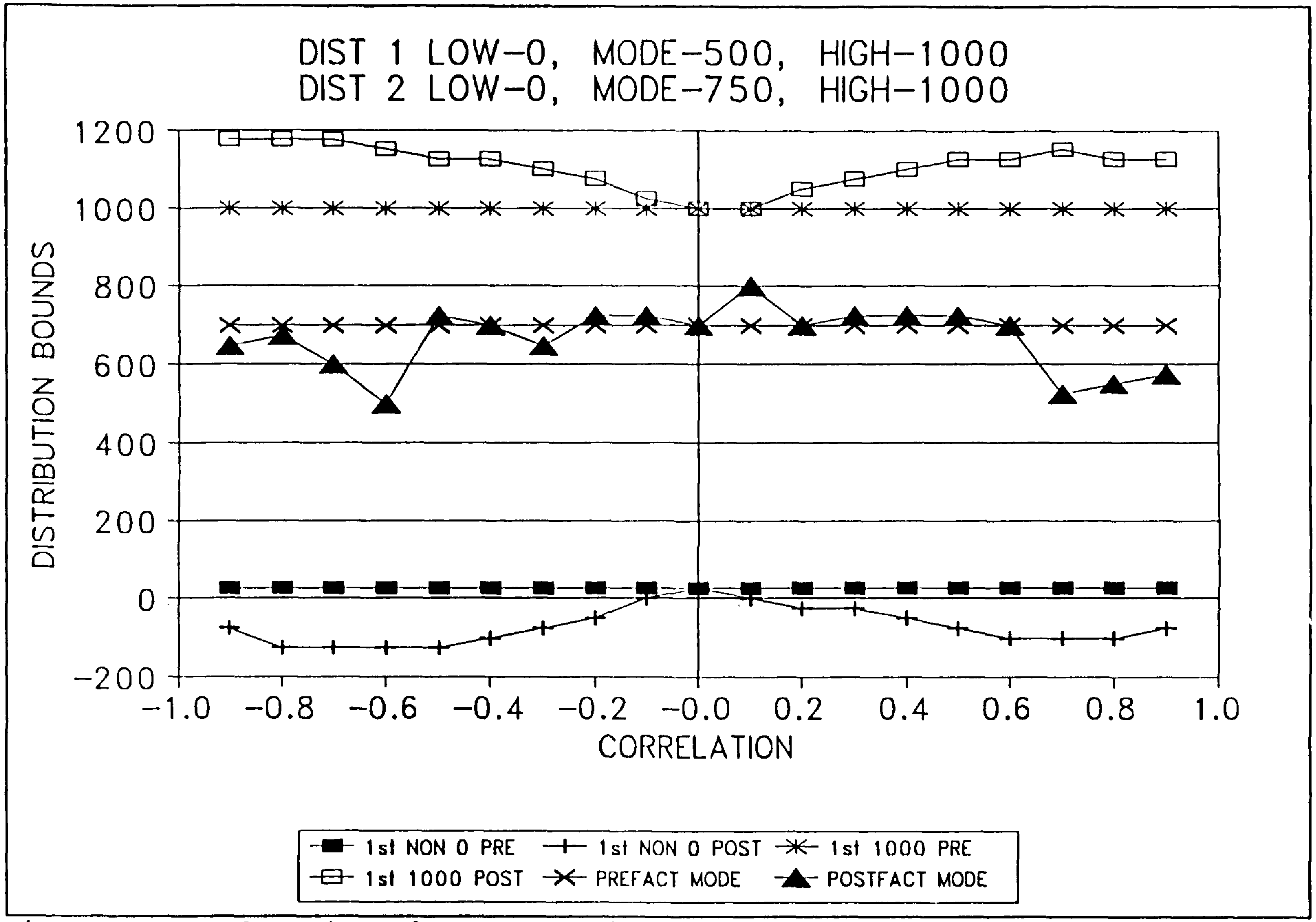


Figure 44 Boundary chart for Case 14 with random seed 1

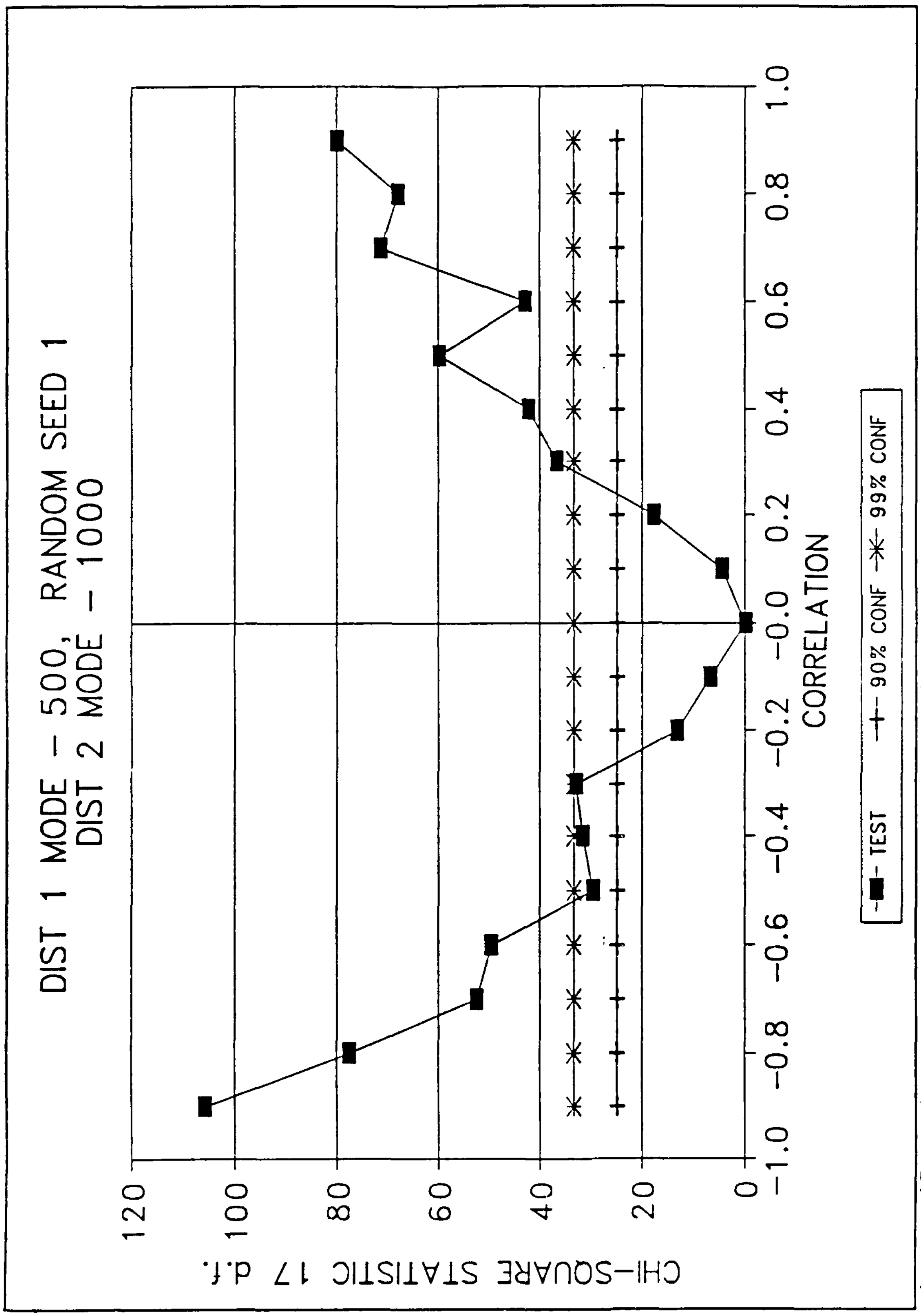


Figure 45 Chi-square test for Case 15 with random seed 1 and 17 d.f.

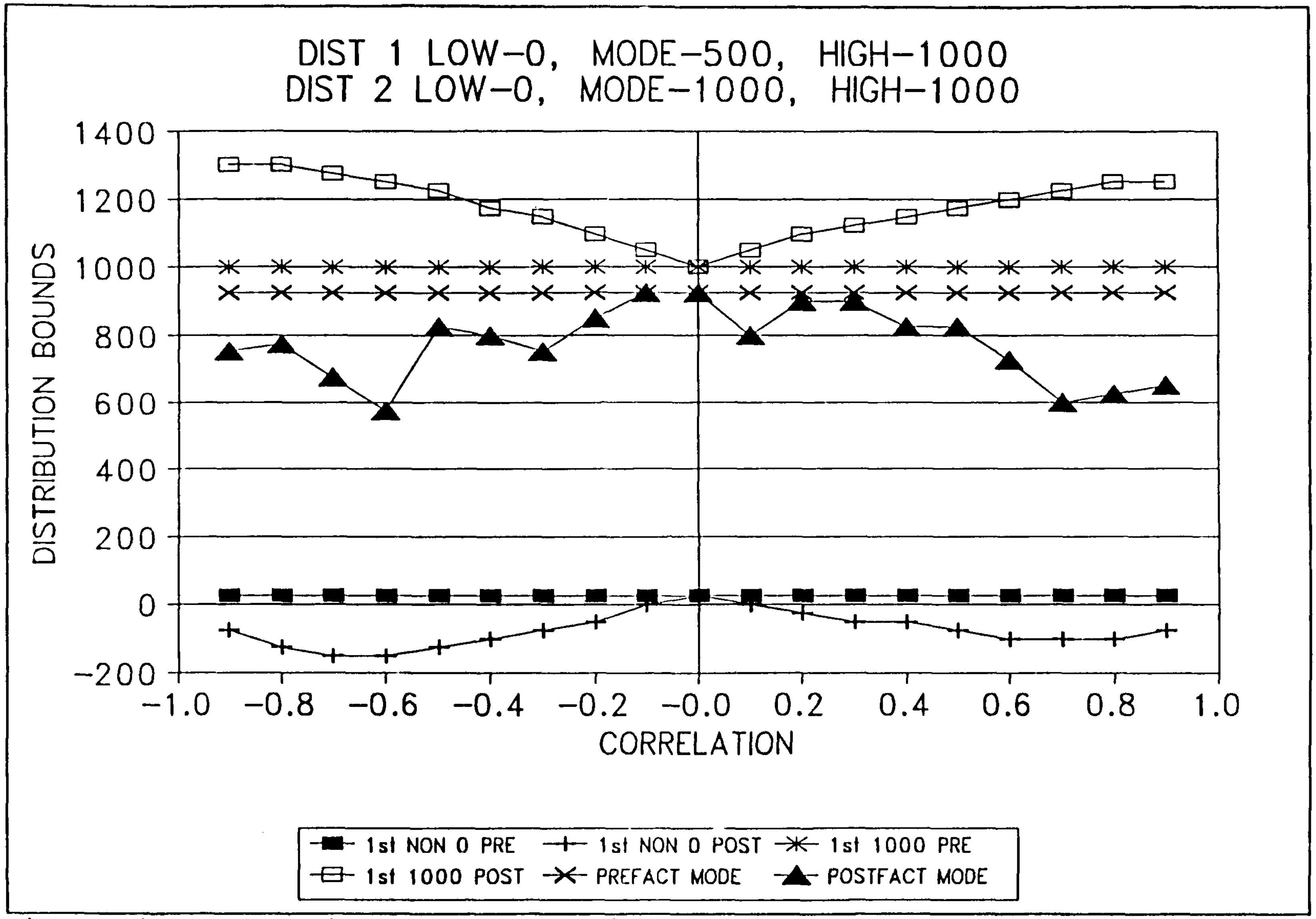


Figure 46 Boundary chart for Case 15 with random seed 1

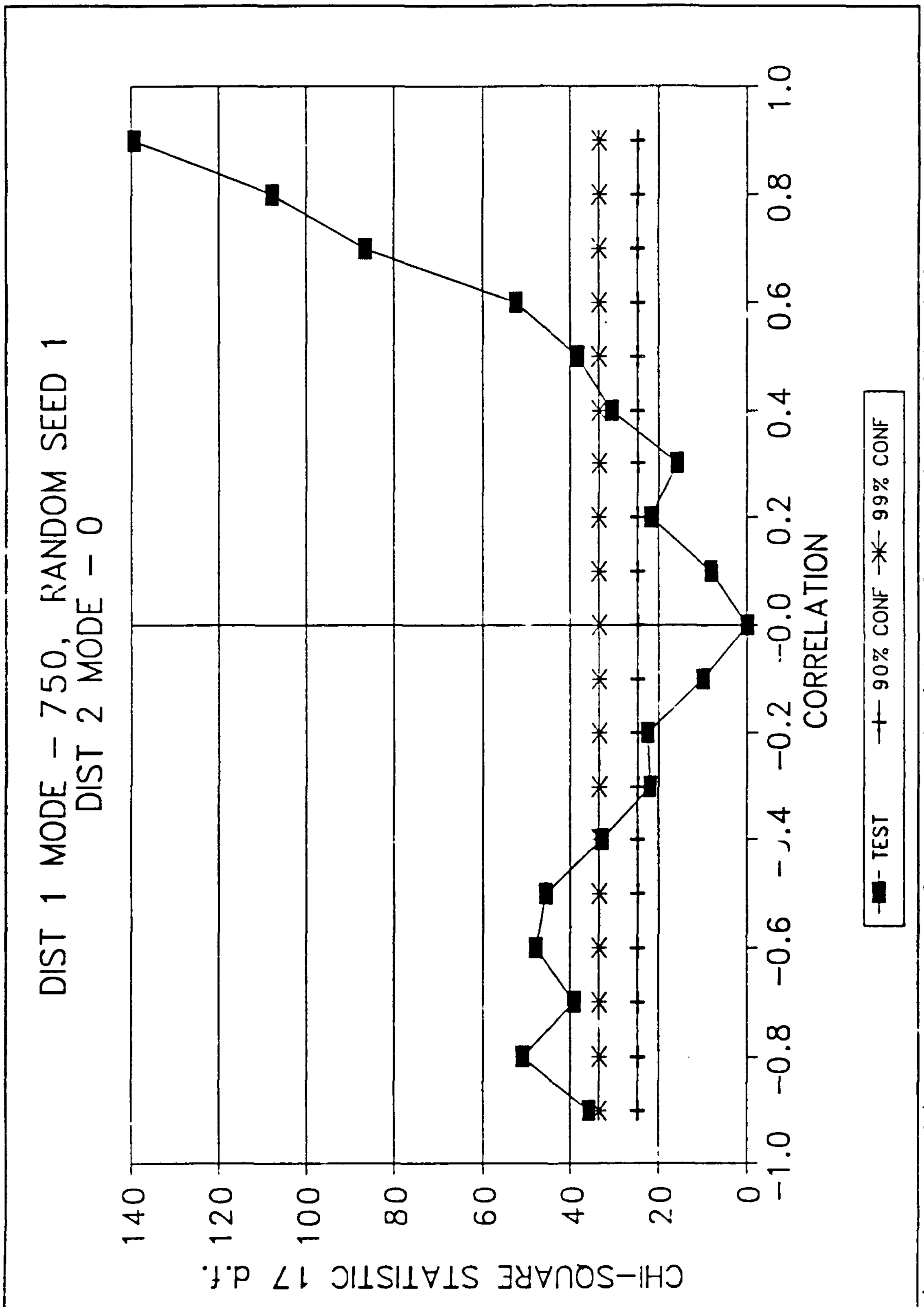


Figure 47 Chi-square test for Case 16 with random seed 1 and 17 d.f.

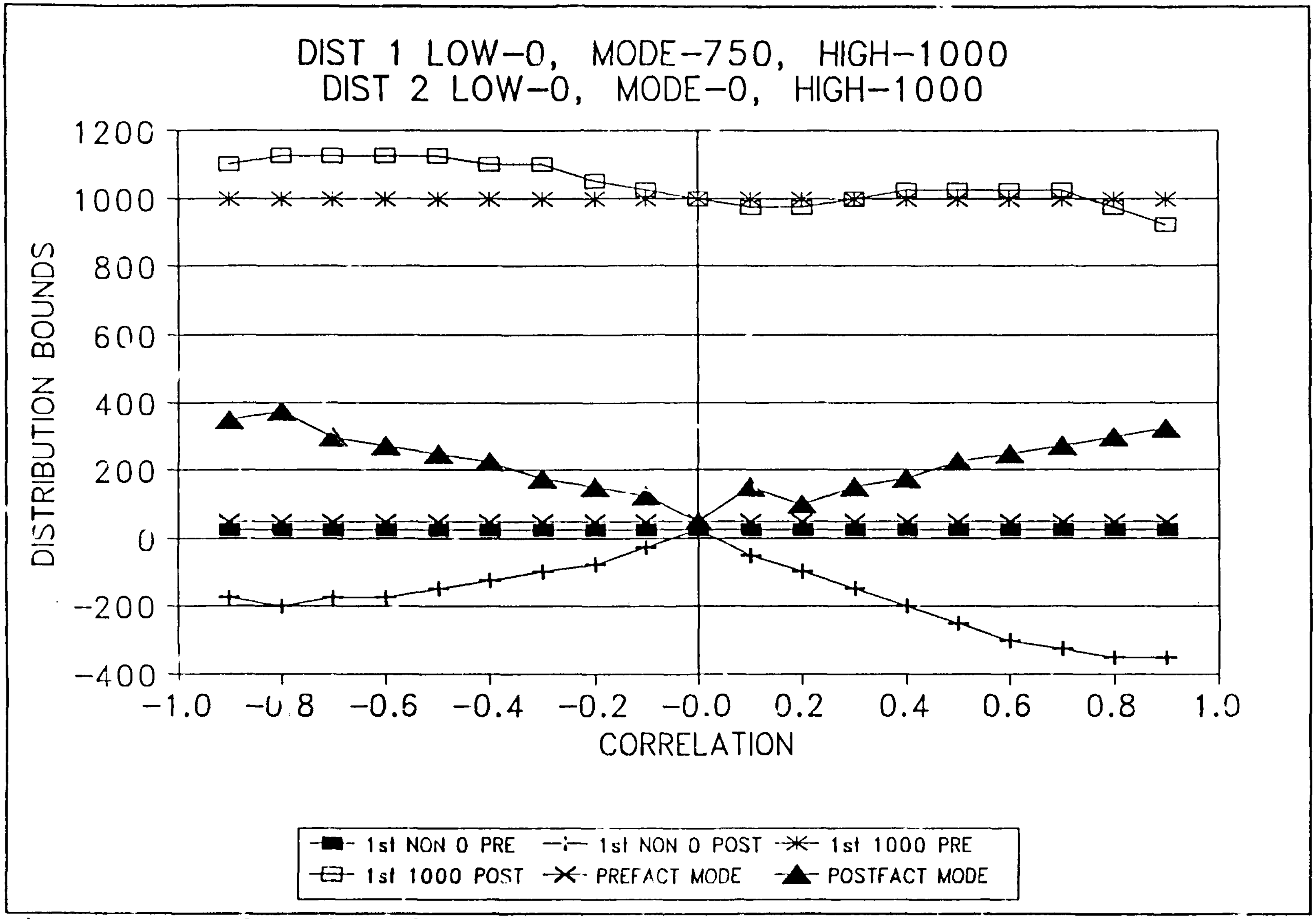


Figure 48 Boundary chart for Case 16 with random seed 1

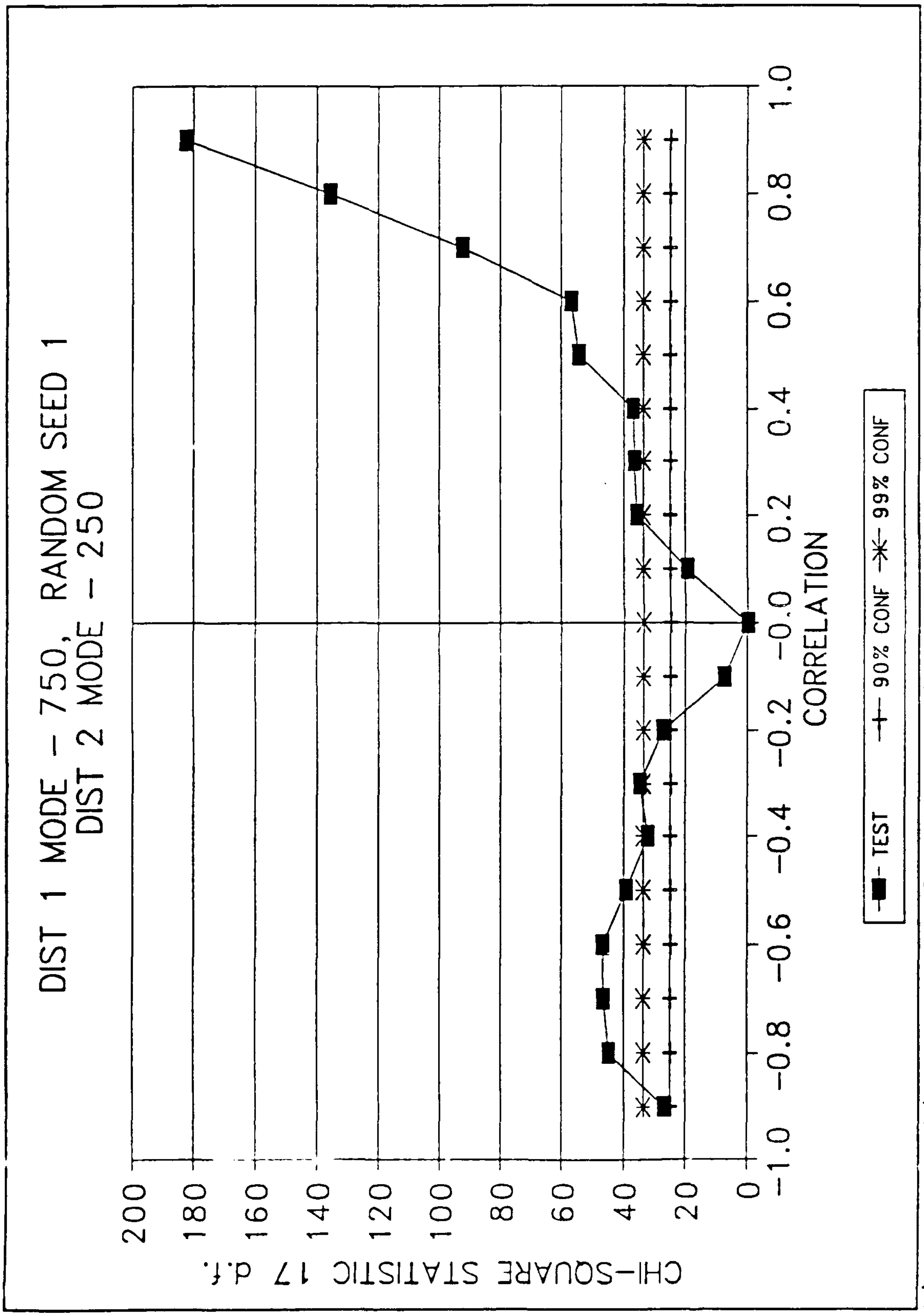


Figure 49 Chi-square test for Case 17 with random seed 1 and 17 d.f.

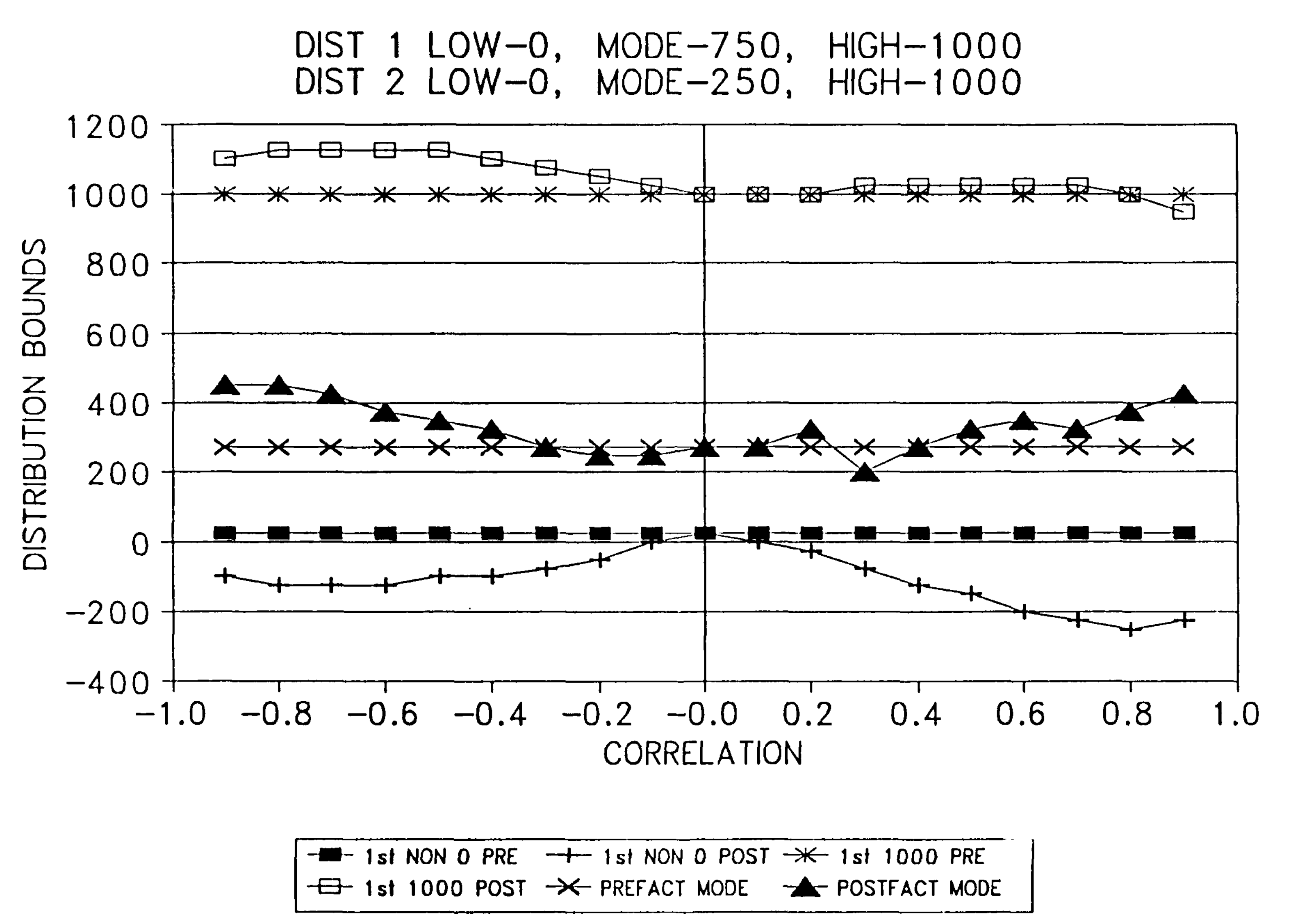


Figure 50 Boundary chart for Case 17 with random seed 1

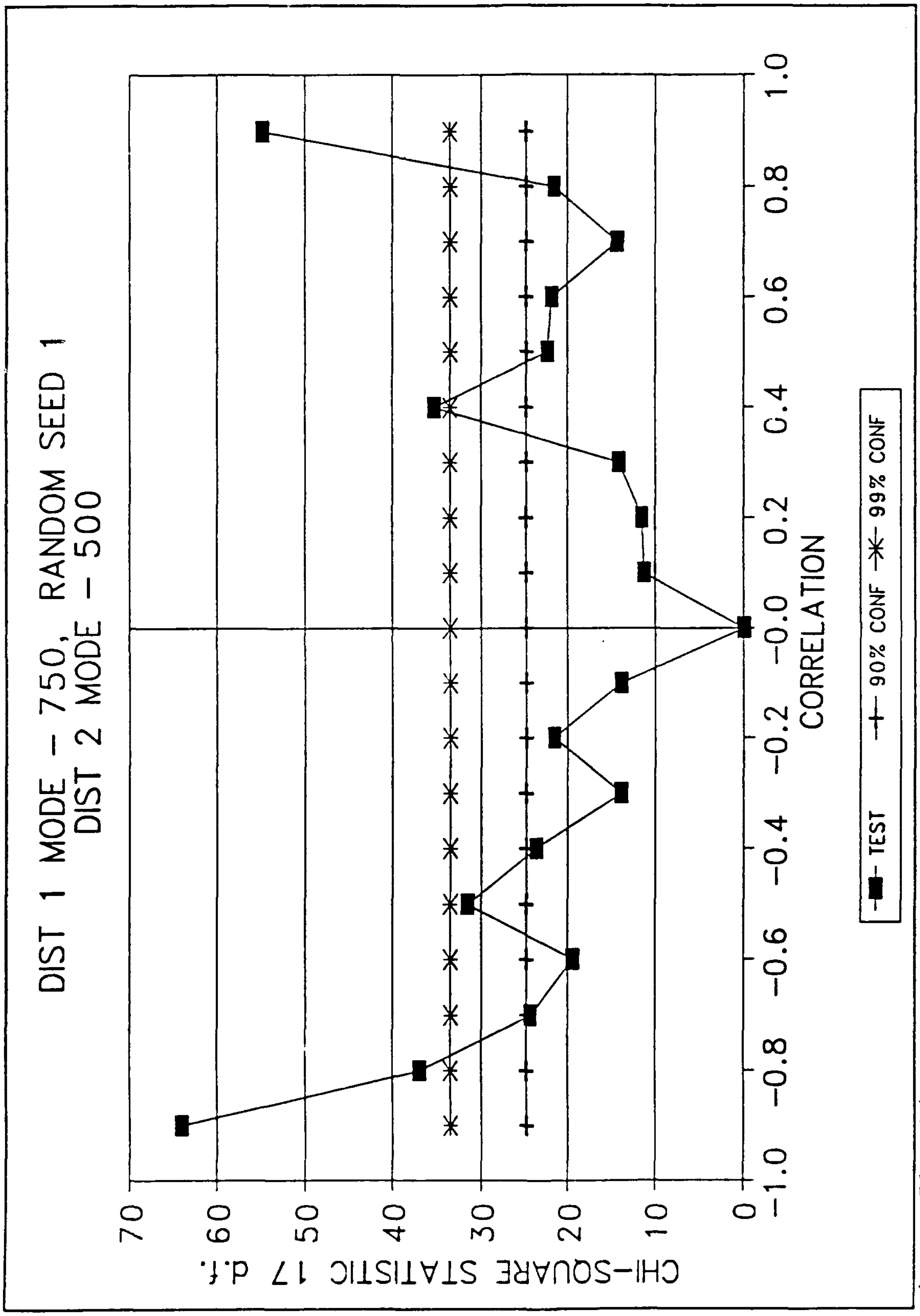


Figure 51 Chi-square test for Case 18 with random seed 1 and 17 d.f.

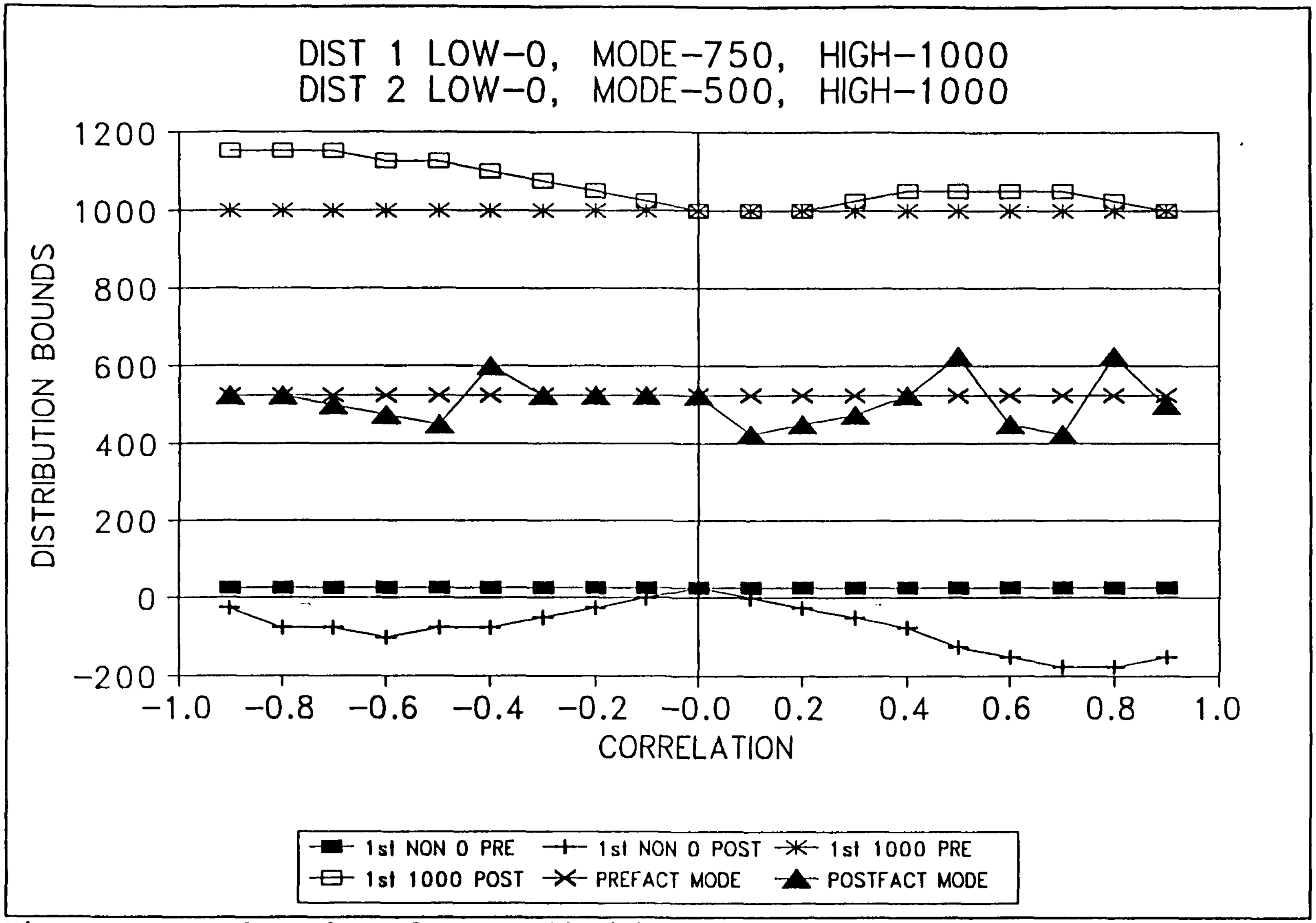


Figure 52 Boundary chart for Case 18 with random seed 1

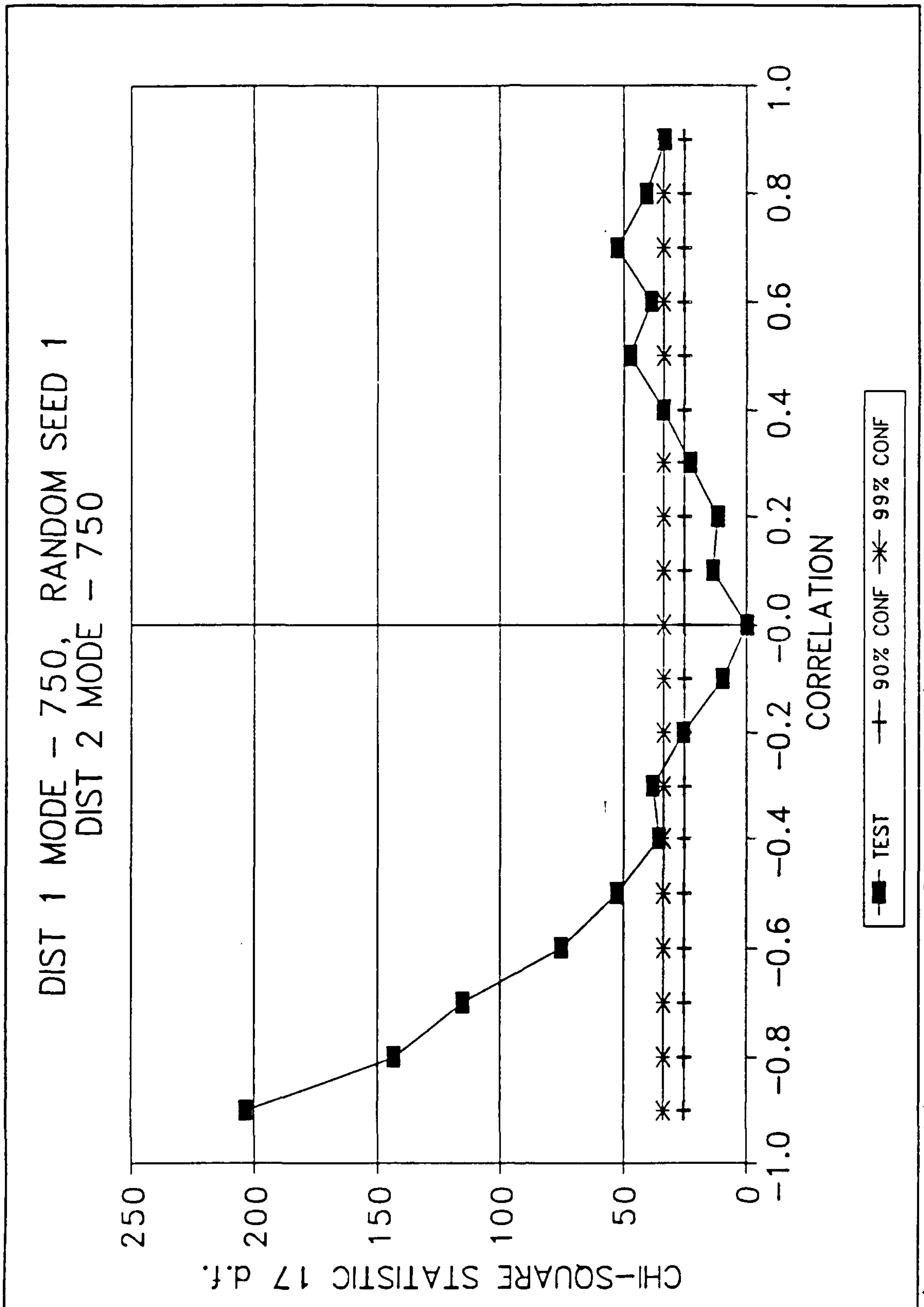


Figure 53 Chi-square test for Case 19 with random seed 1 and 17 d.f.

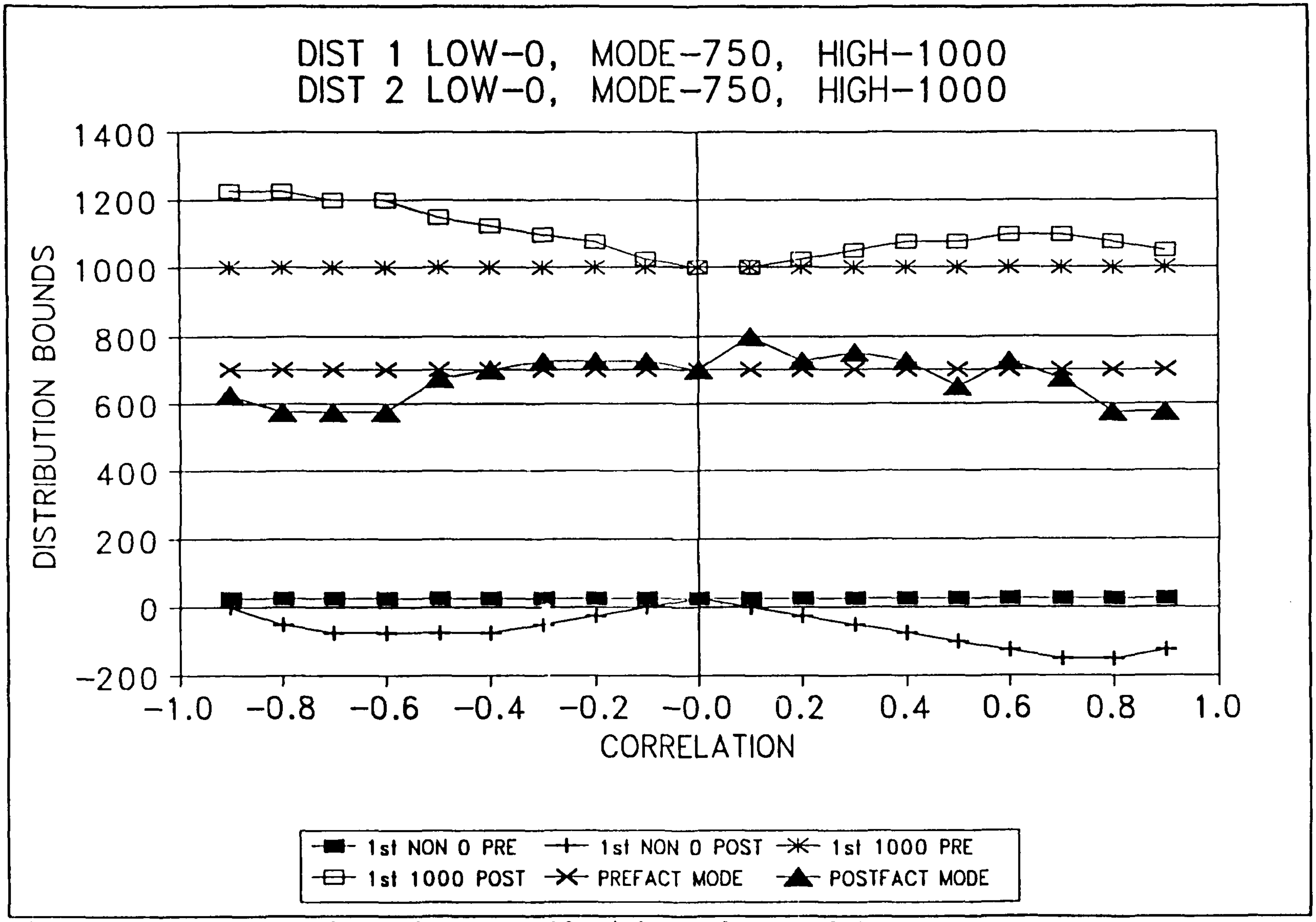


Figure 54 Boundary chart for Case 19 with random seed 1

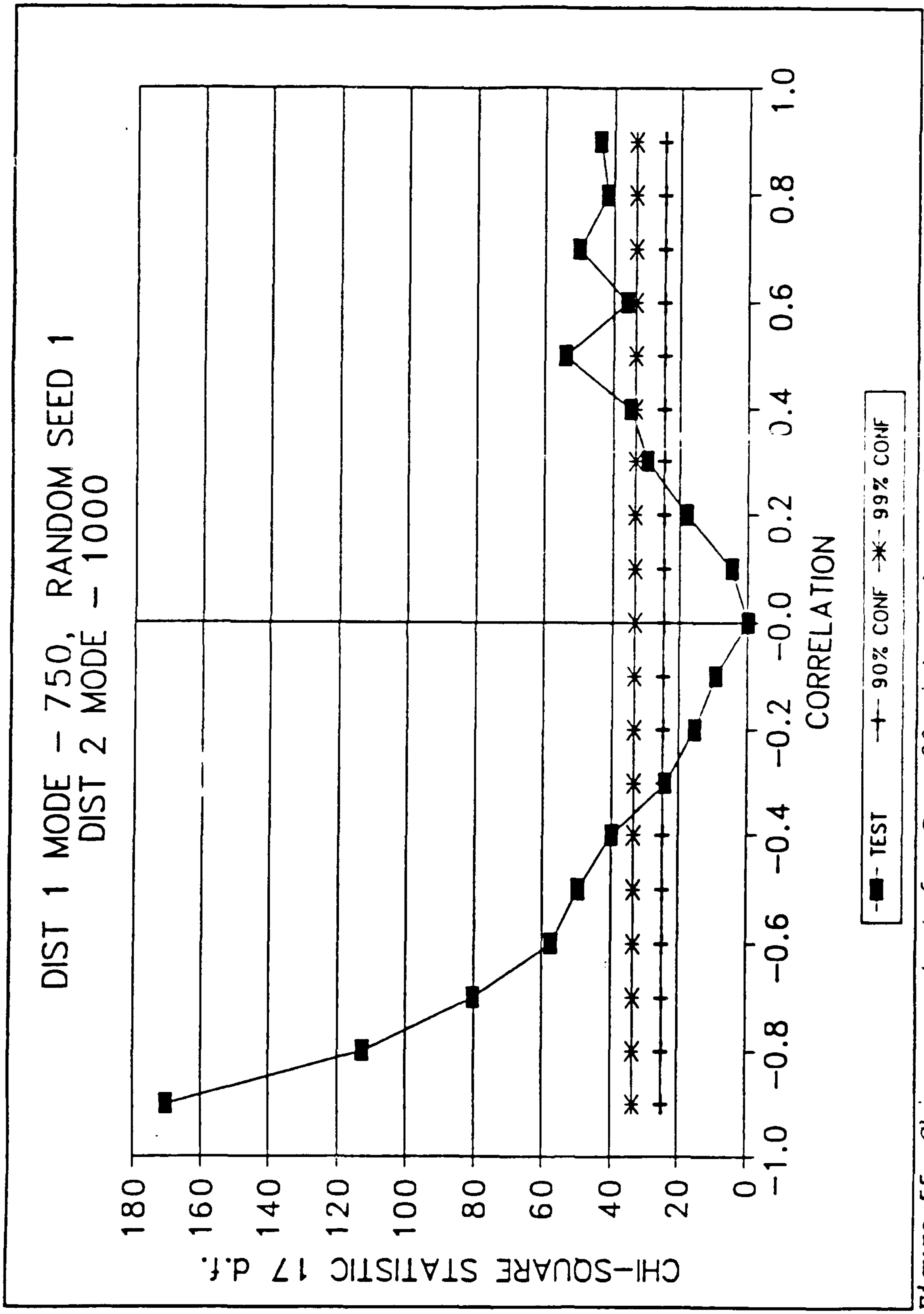


Figure 55 Chi-square test for Case 20 with random seed 2 and 17 d.f.

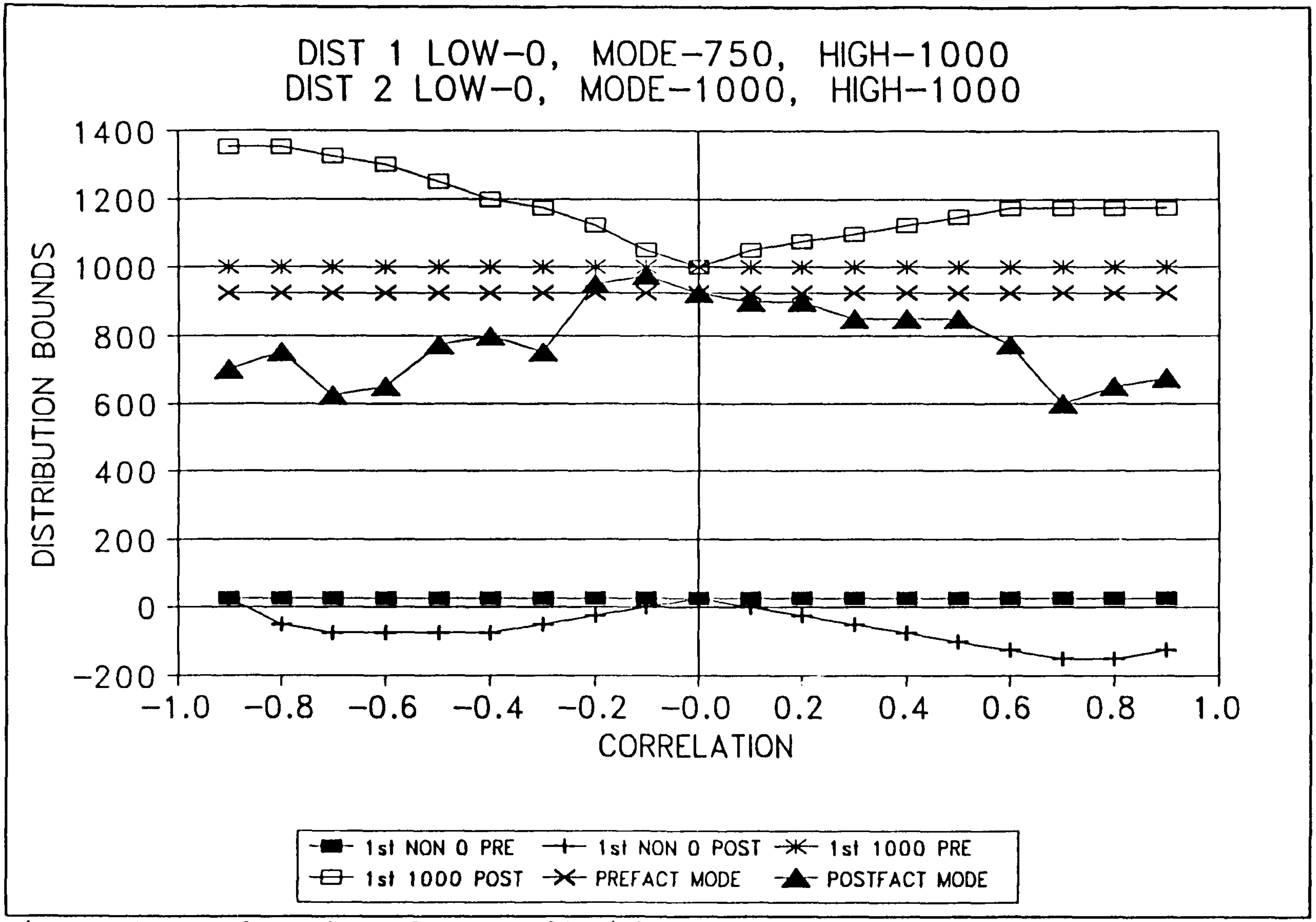


Figure 56 Boundary chart for Case 20 with random seed 1

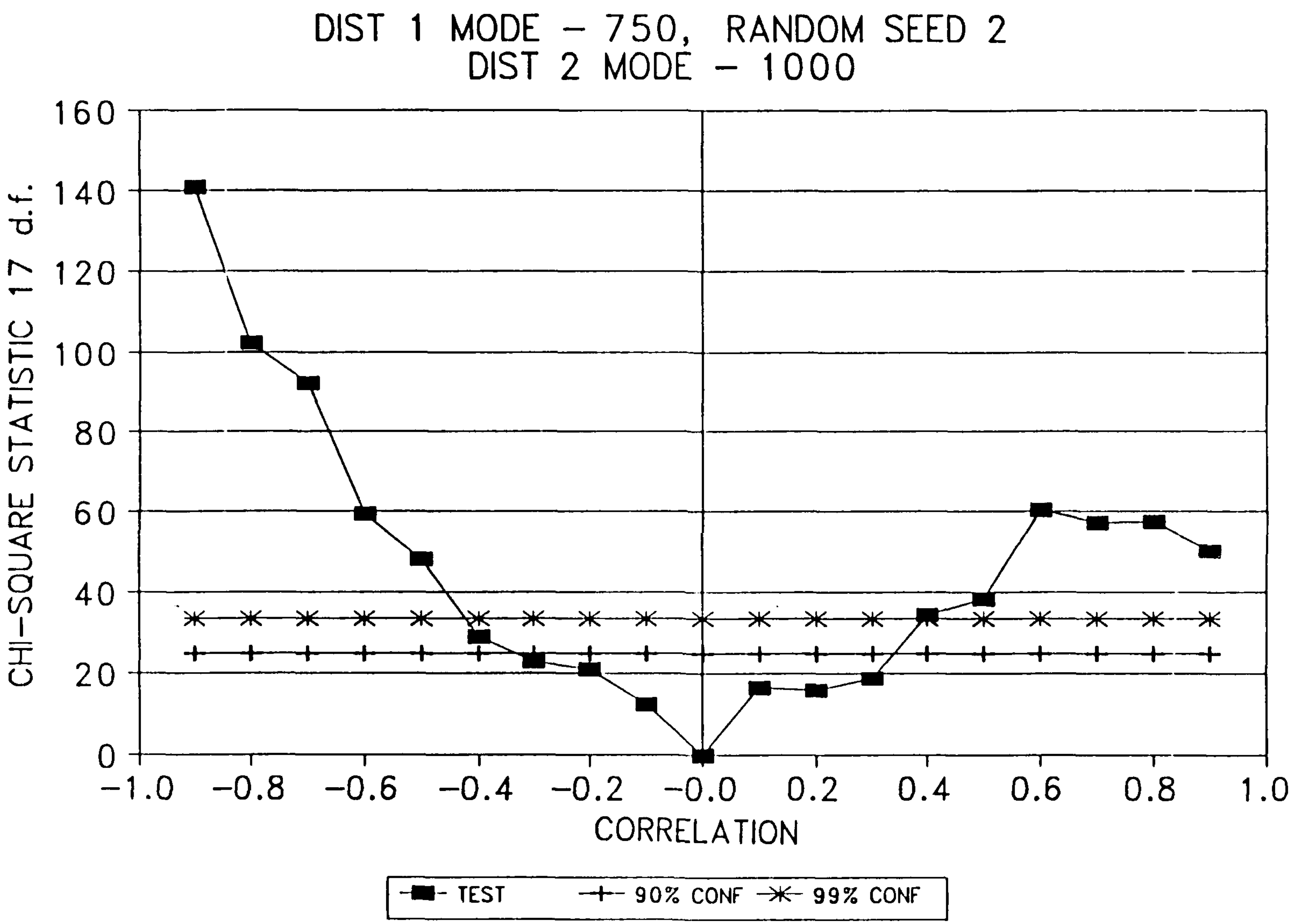


Figure 57 Chi-square test for Case 20 with random seed 2 and 17 d.f.

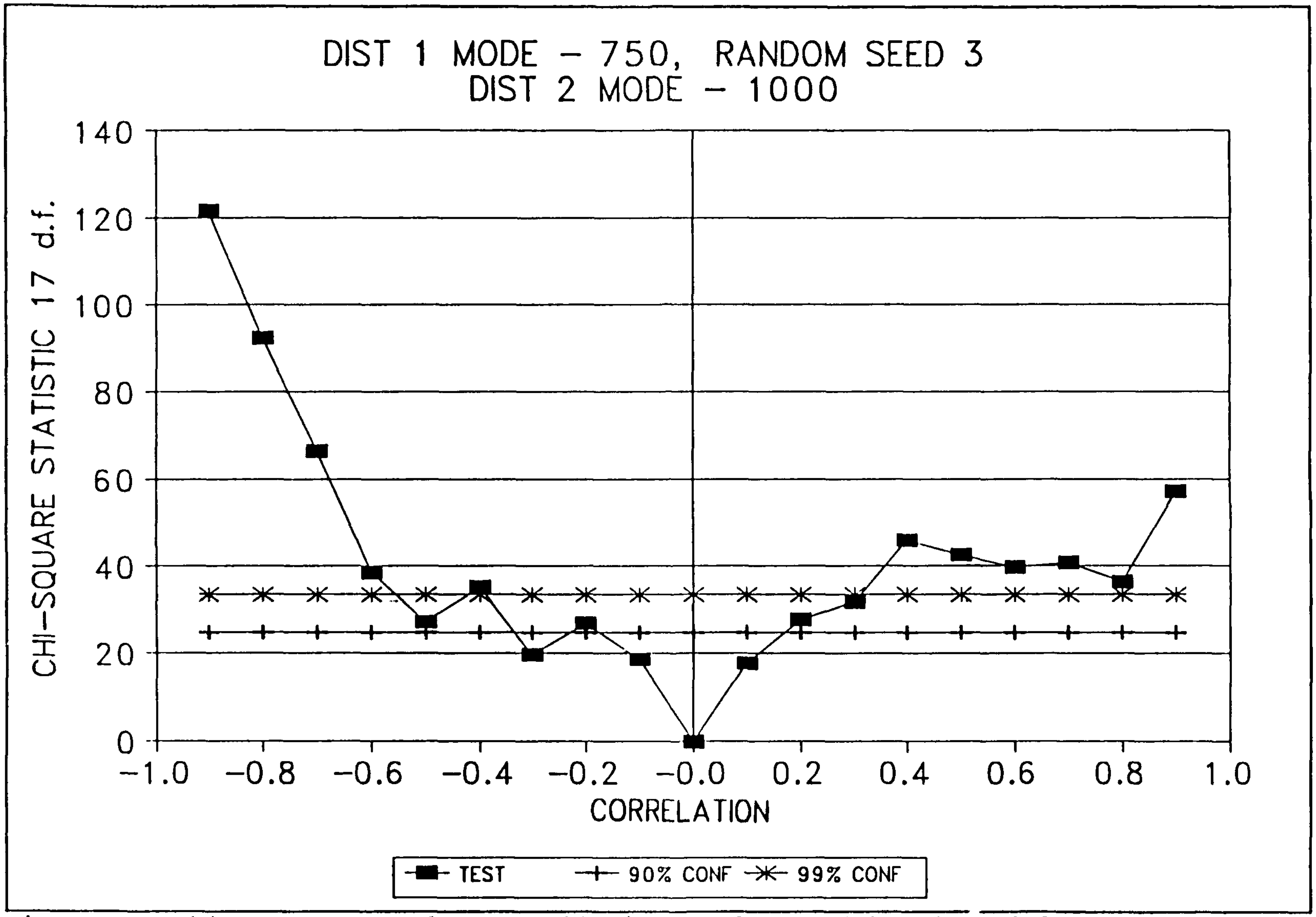


Figure 58 Chi-square test for Case 20 with random seed 3 and 17 d.f.

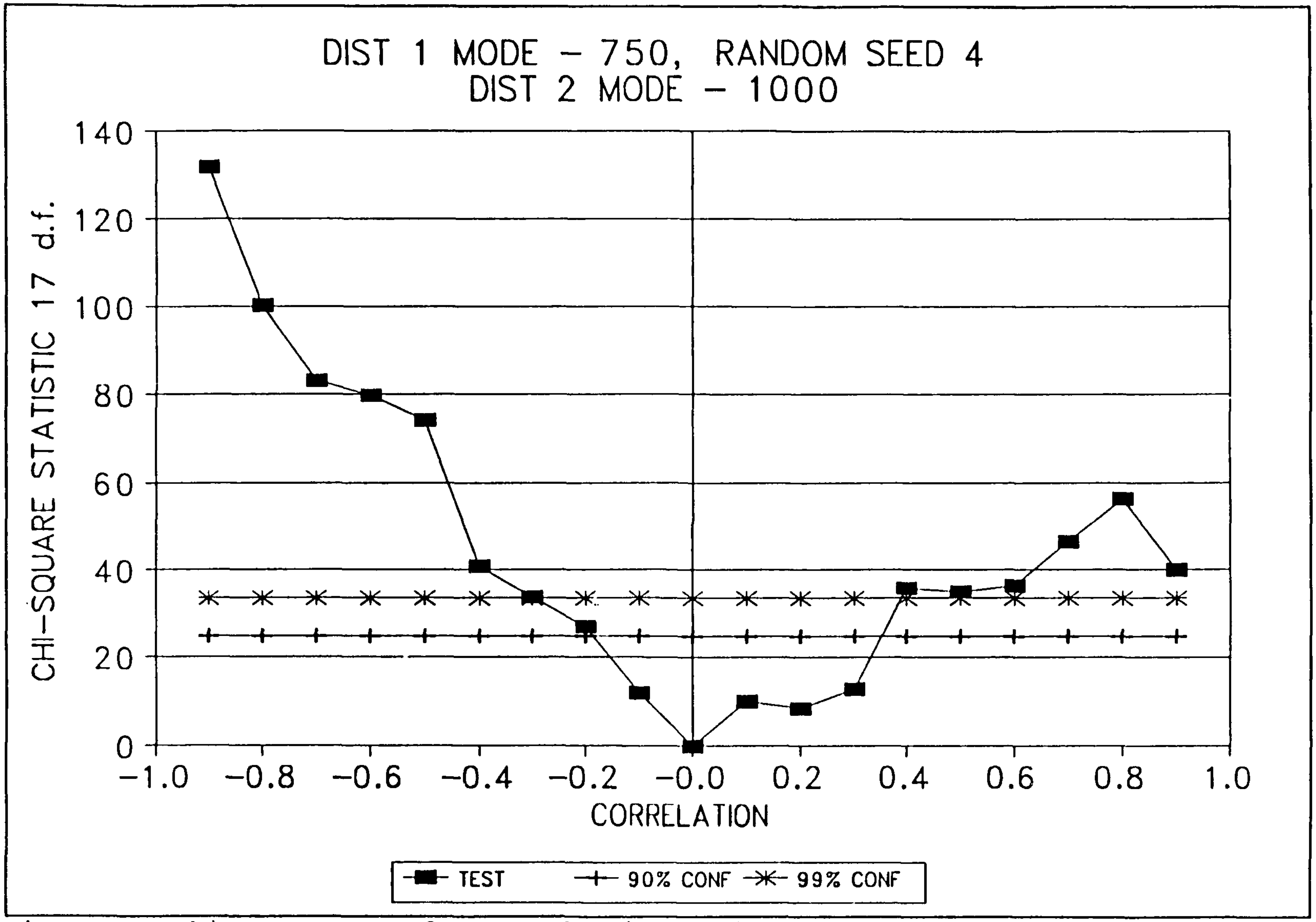


Figure 59 Chi-square test for Case 20 with random seed 4 and 17 d.f.

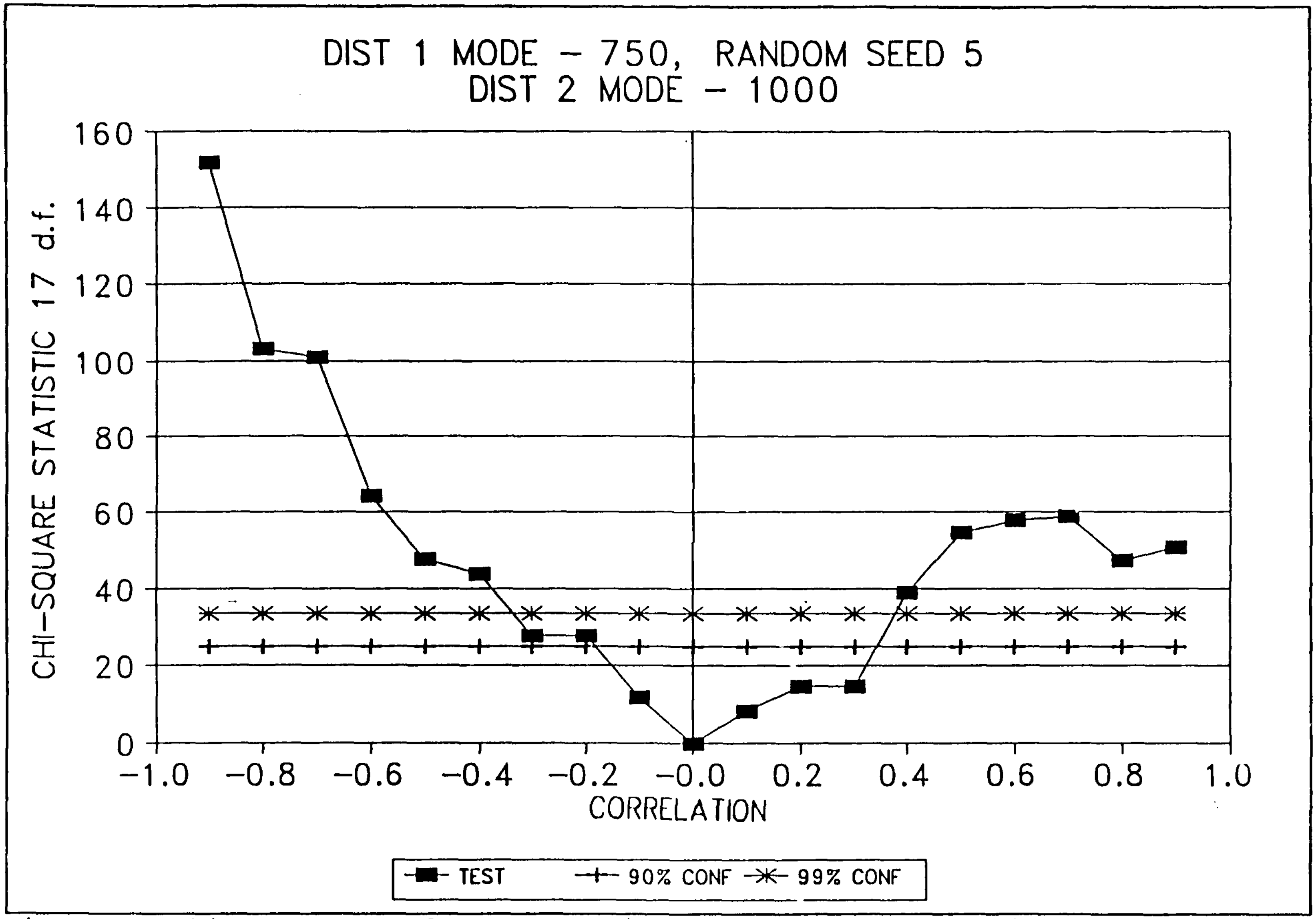


Figure 60 Chi-square test for Case 20 with random seed 5 and 17 d.f.

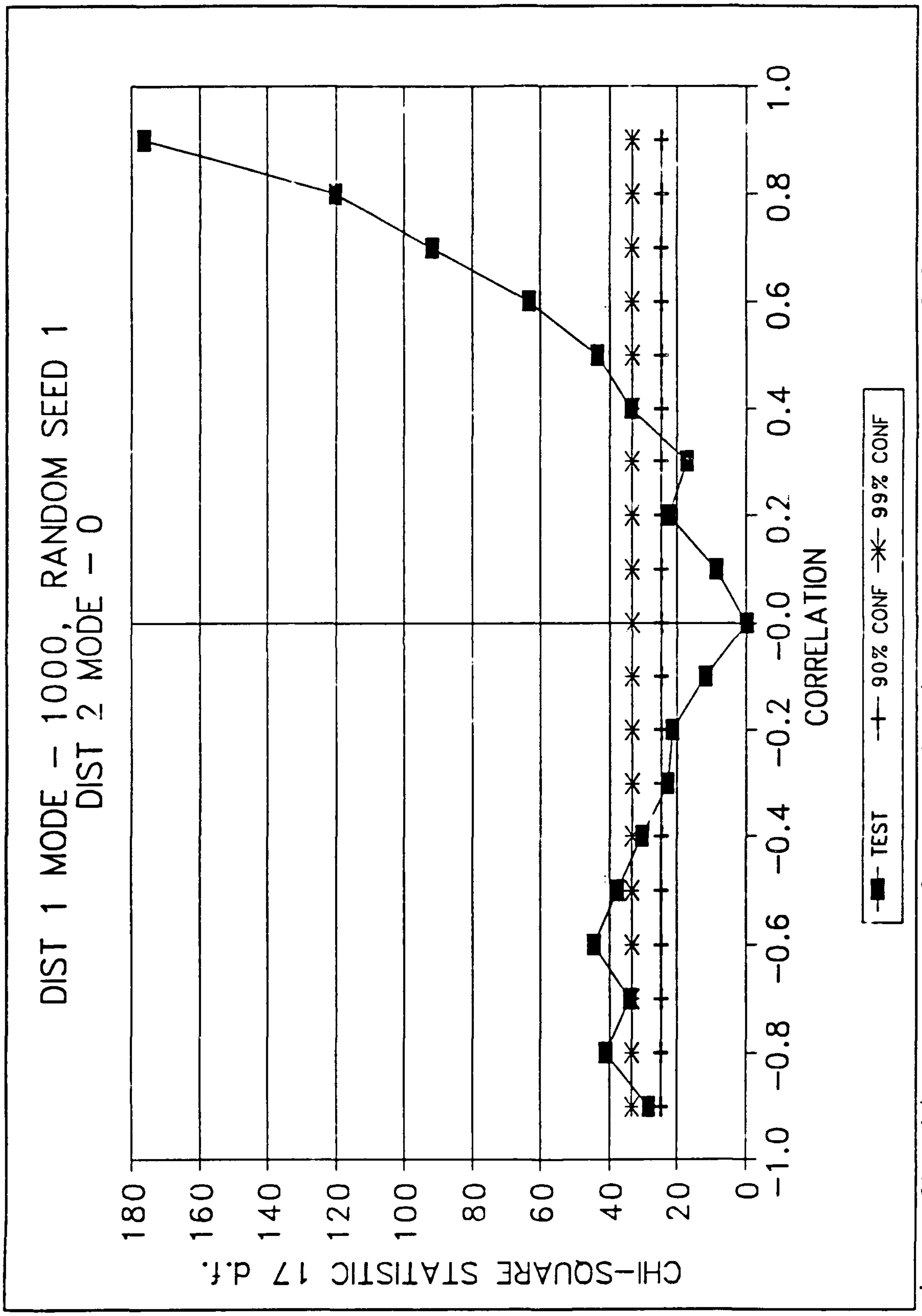


Figure 61 Chi-square test for Case 21 with random seed 1 and 17 d.f.

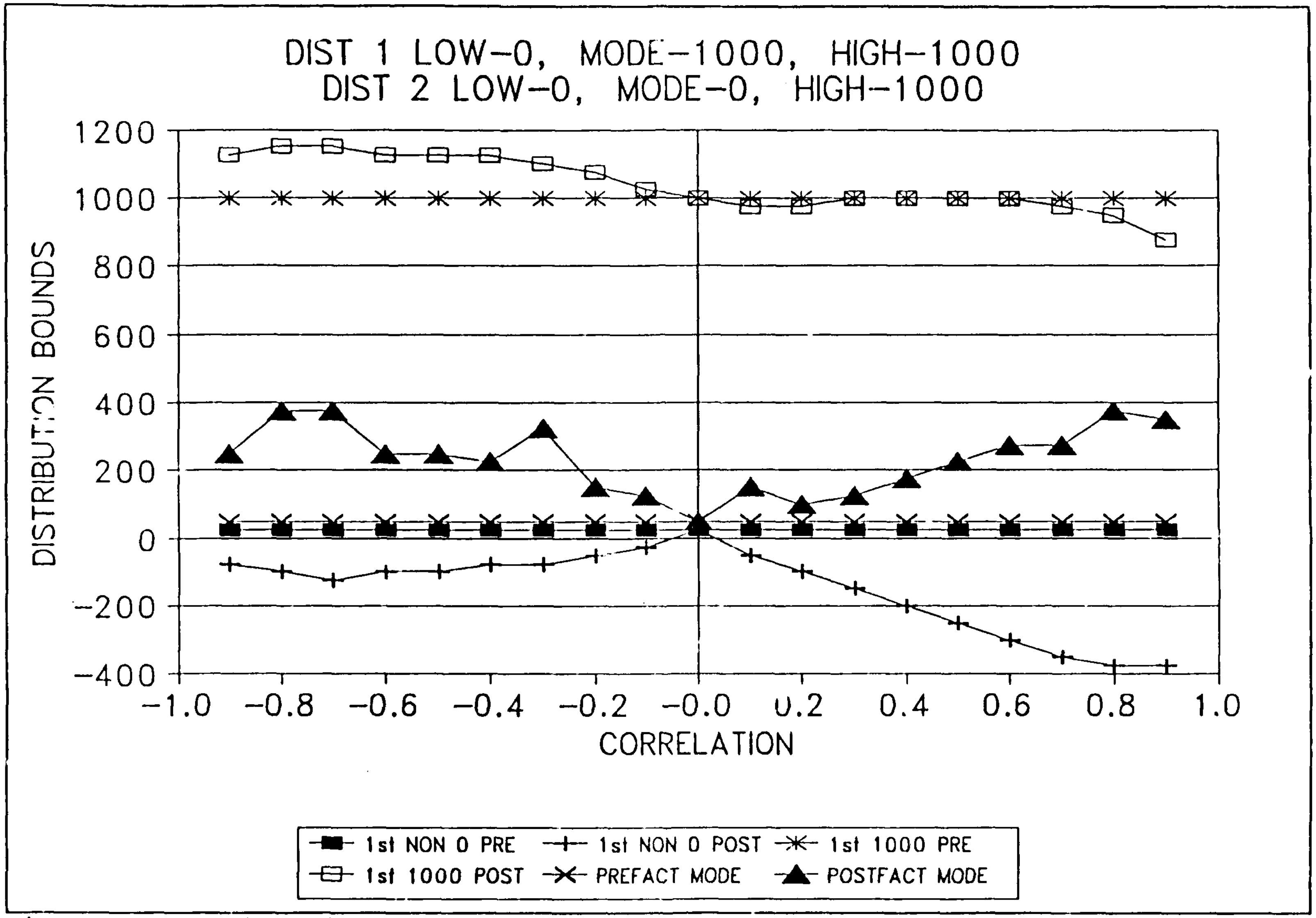


Figure 62 Boundary chart for Case 21 with random seed 1

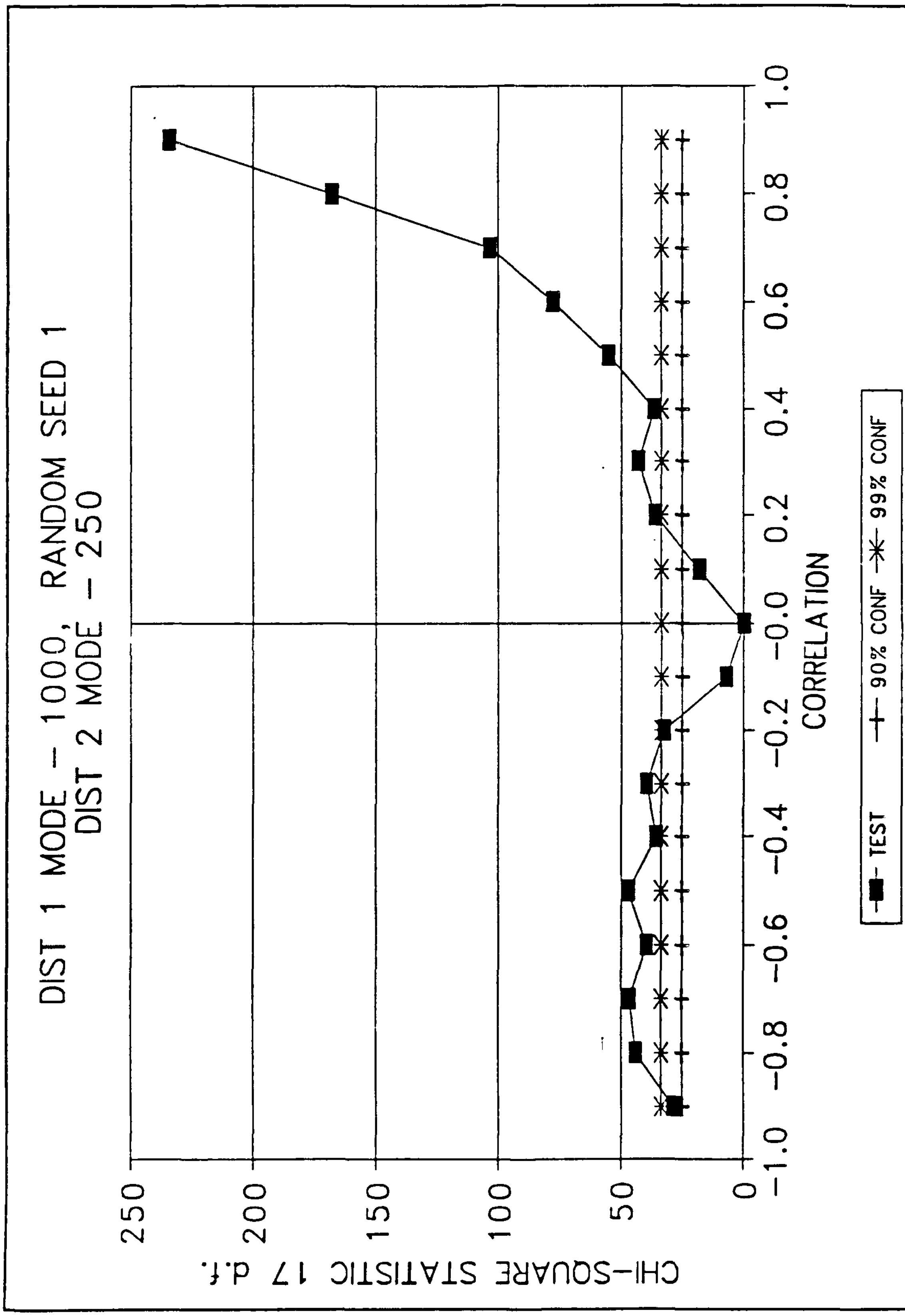


Figure 63 Chi-square test for Case 22 with random seed 1 and 17 d.f.

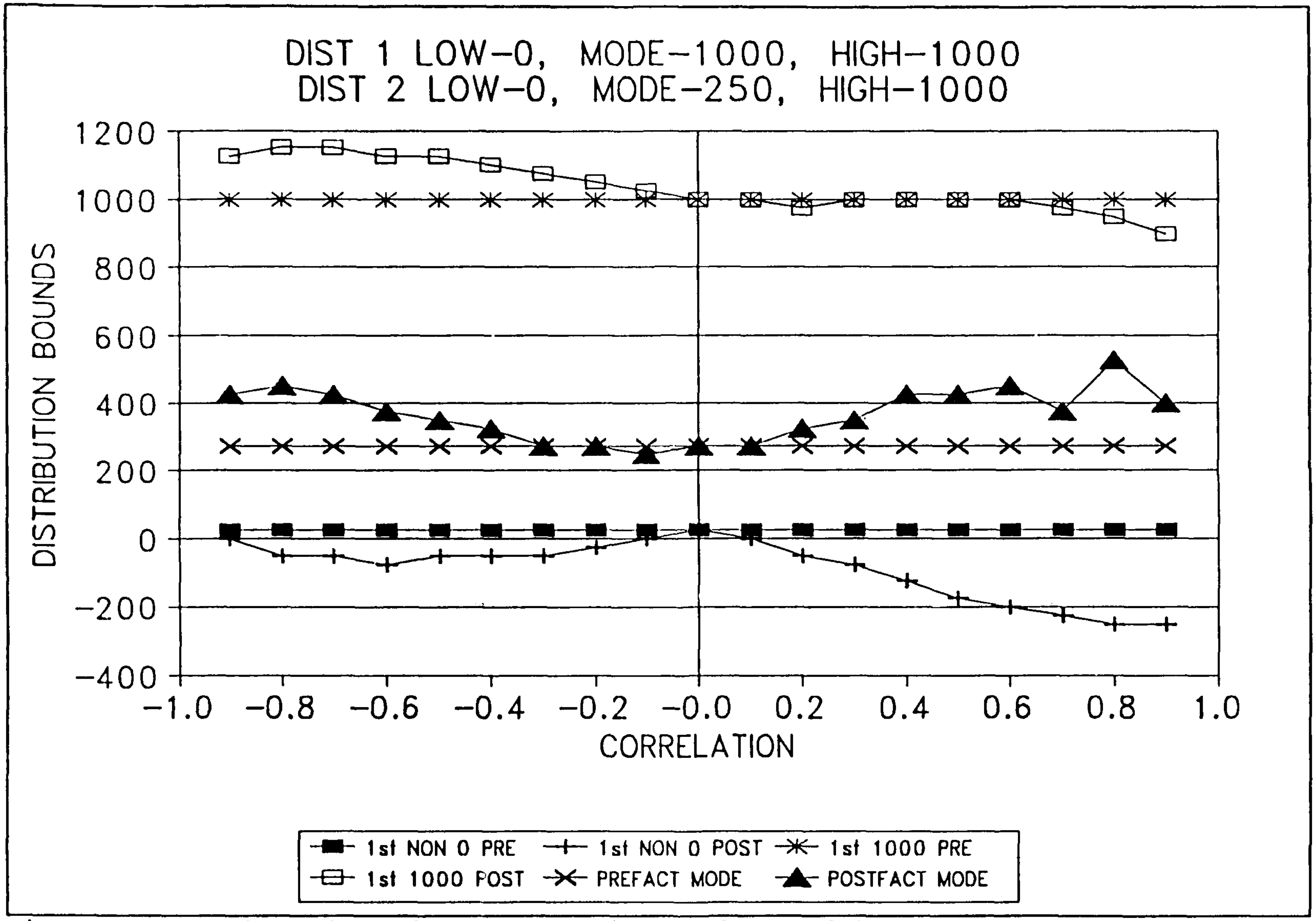


Figure 64 Boundary chart for Case 22 with random seed 1

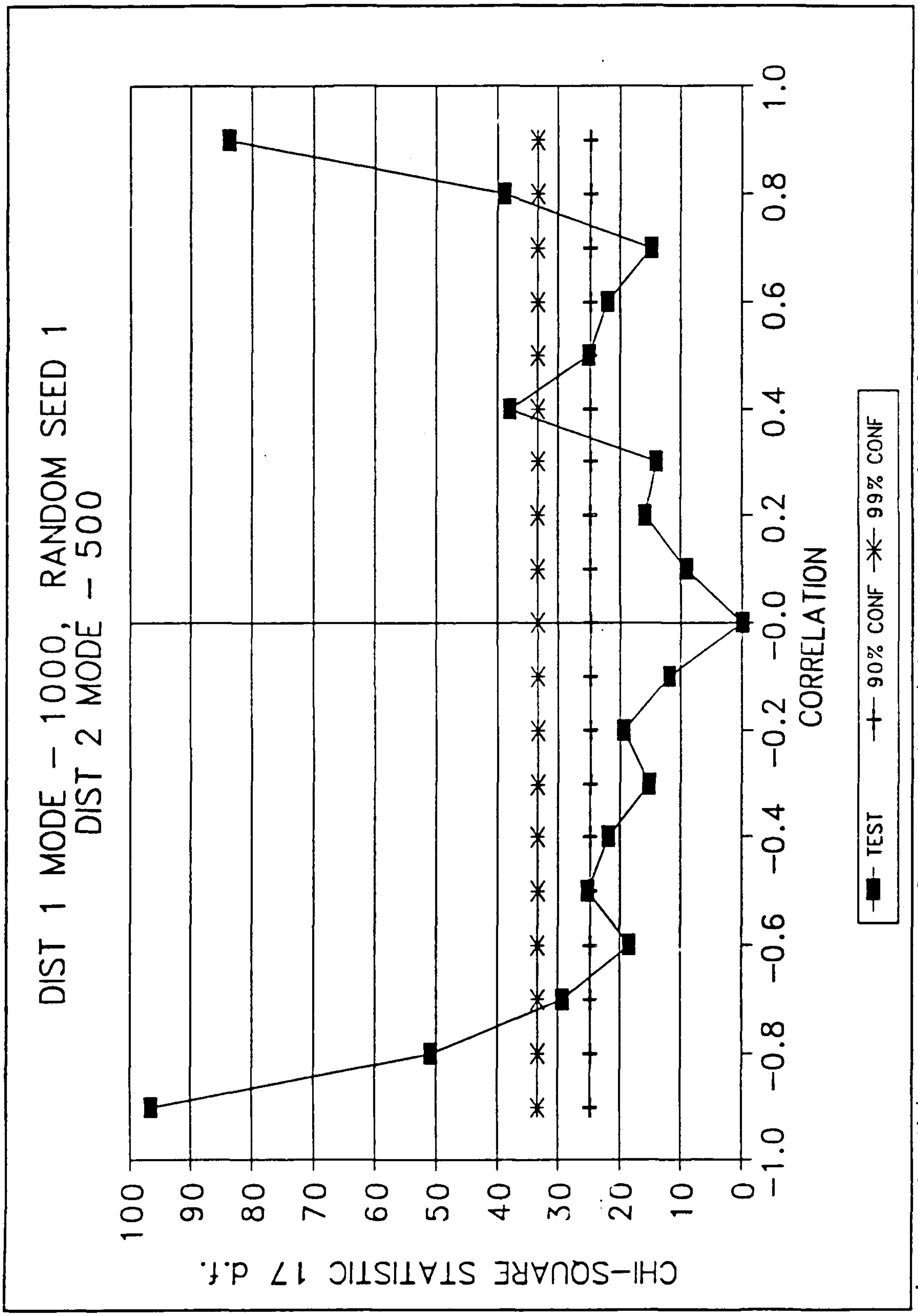


Figure 65 Chi-square test for Case 23 with random seed 1 and 17 d.f.

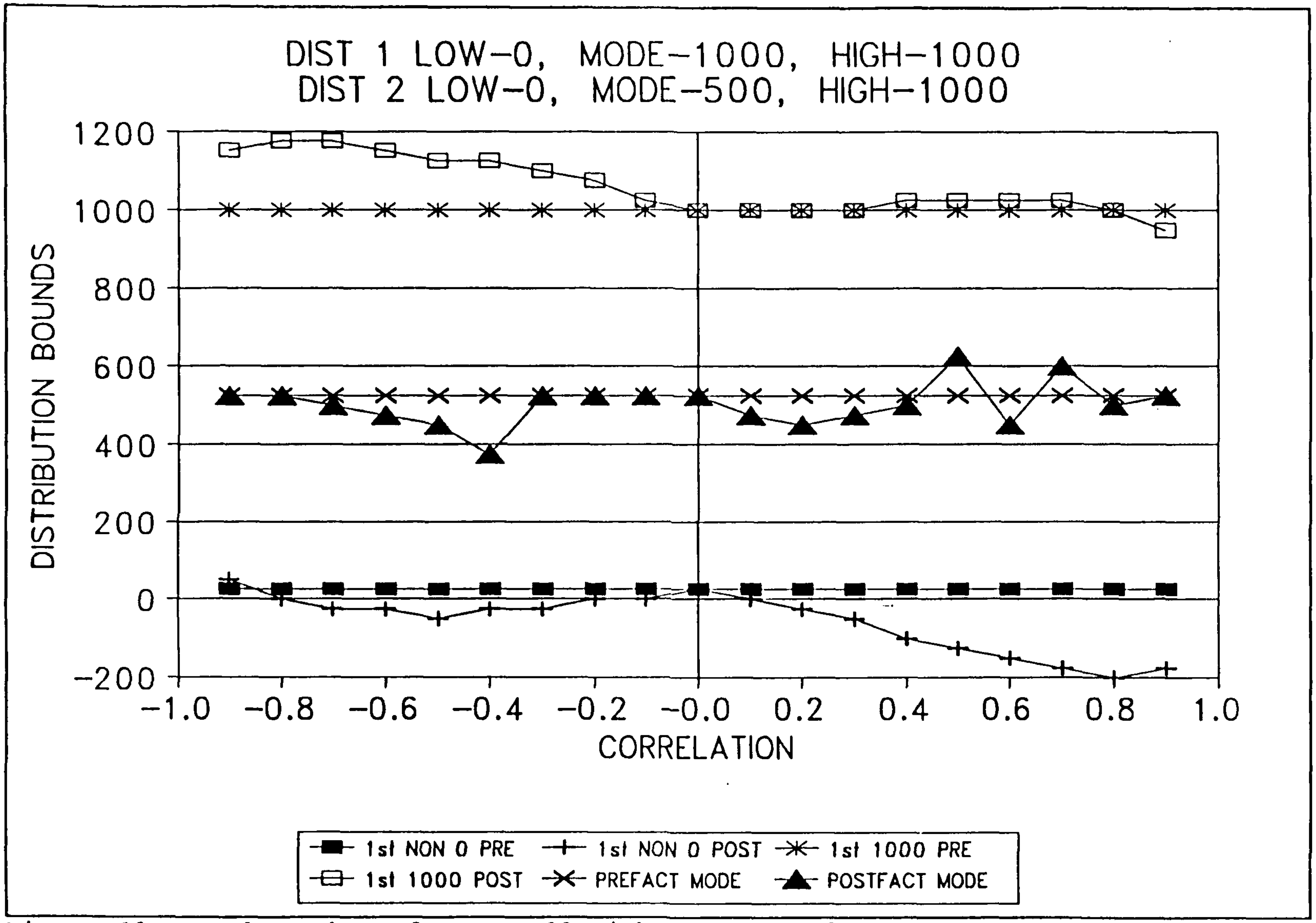


Figure 66 Boundary chart for Case 23 with random seed 1

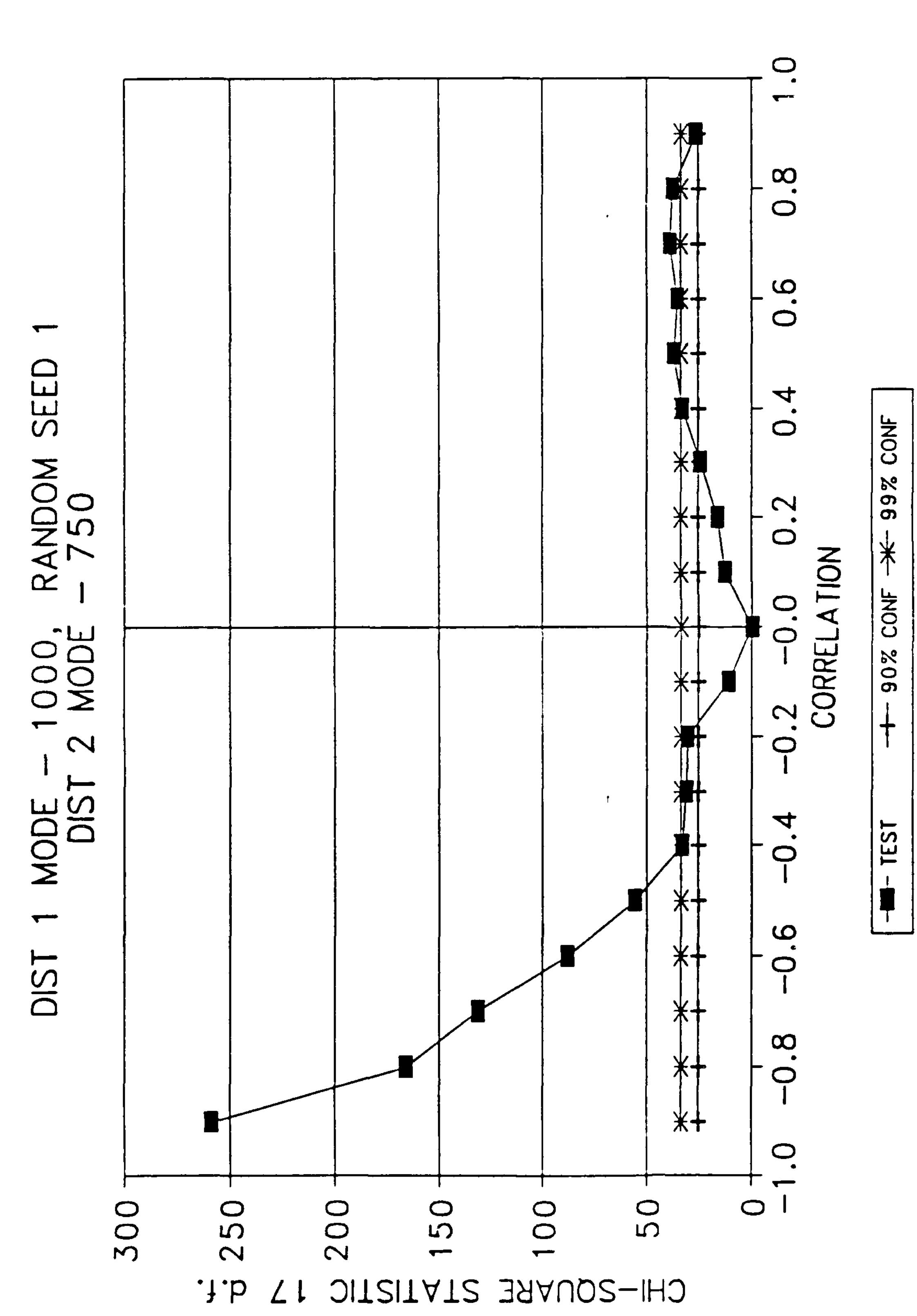


Figure 67 Chi-square test for Case 24 with random seed 1 and 17 d.f.

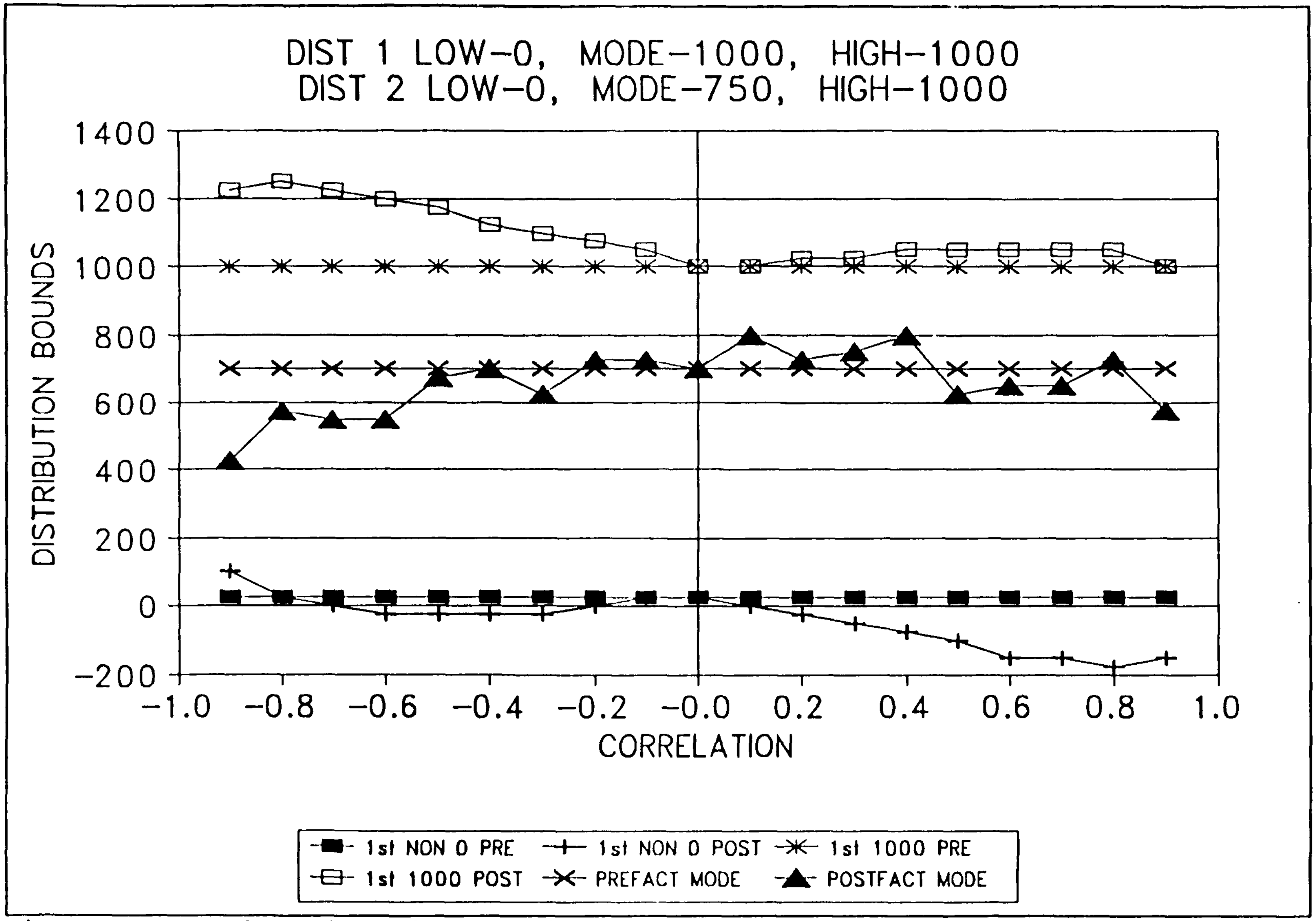


Figure 68 Boundary chart for Case 24 with random seed 1

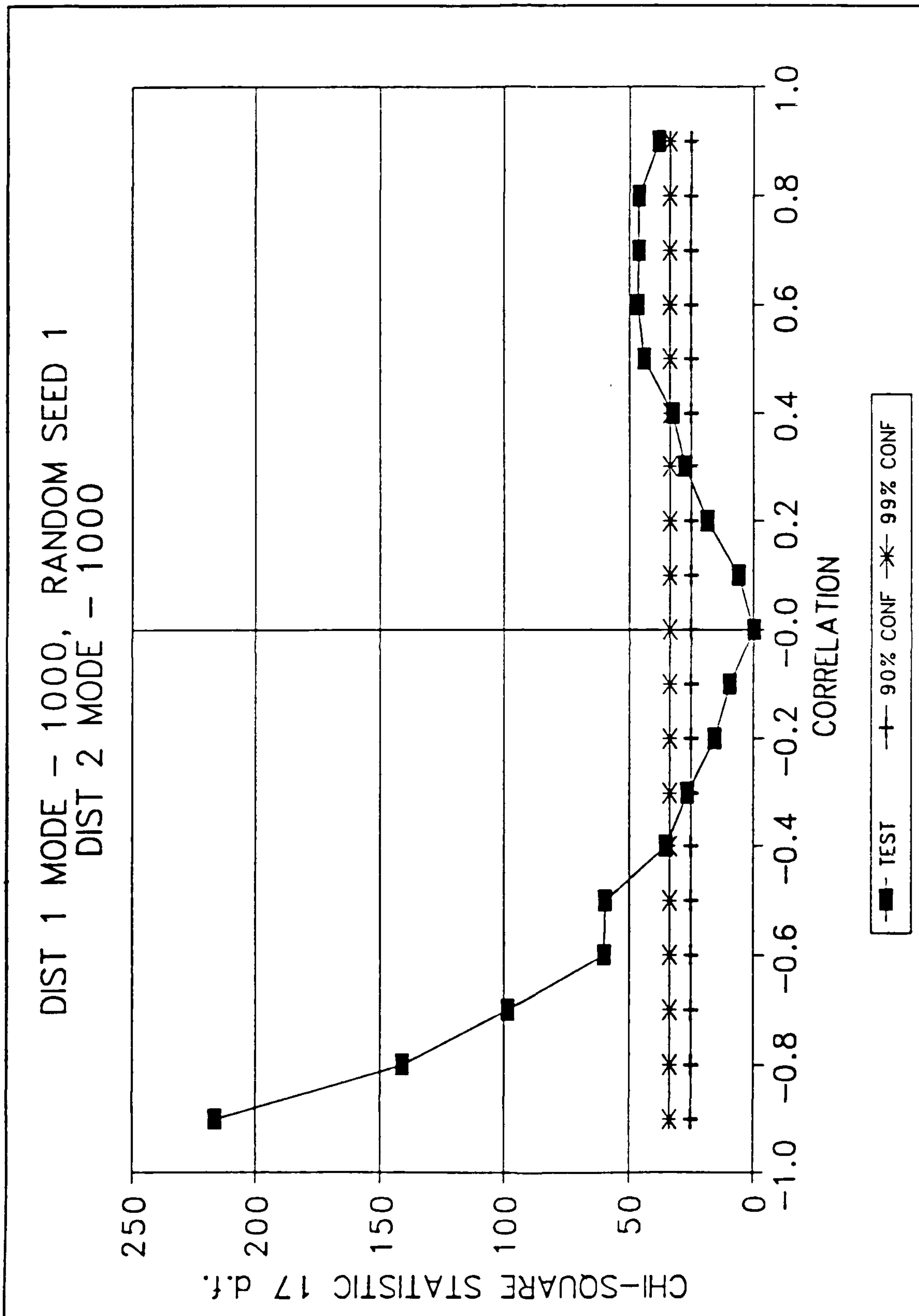


Figure 69 Chi-square test for Case 25 with random seed 1 and 17 d.f.

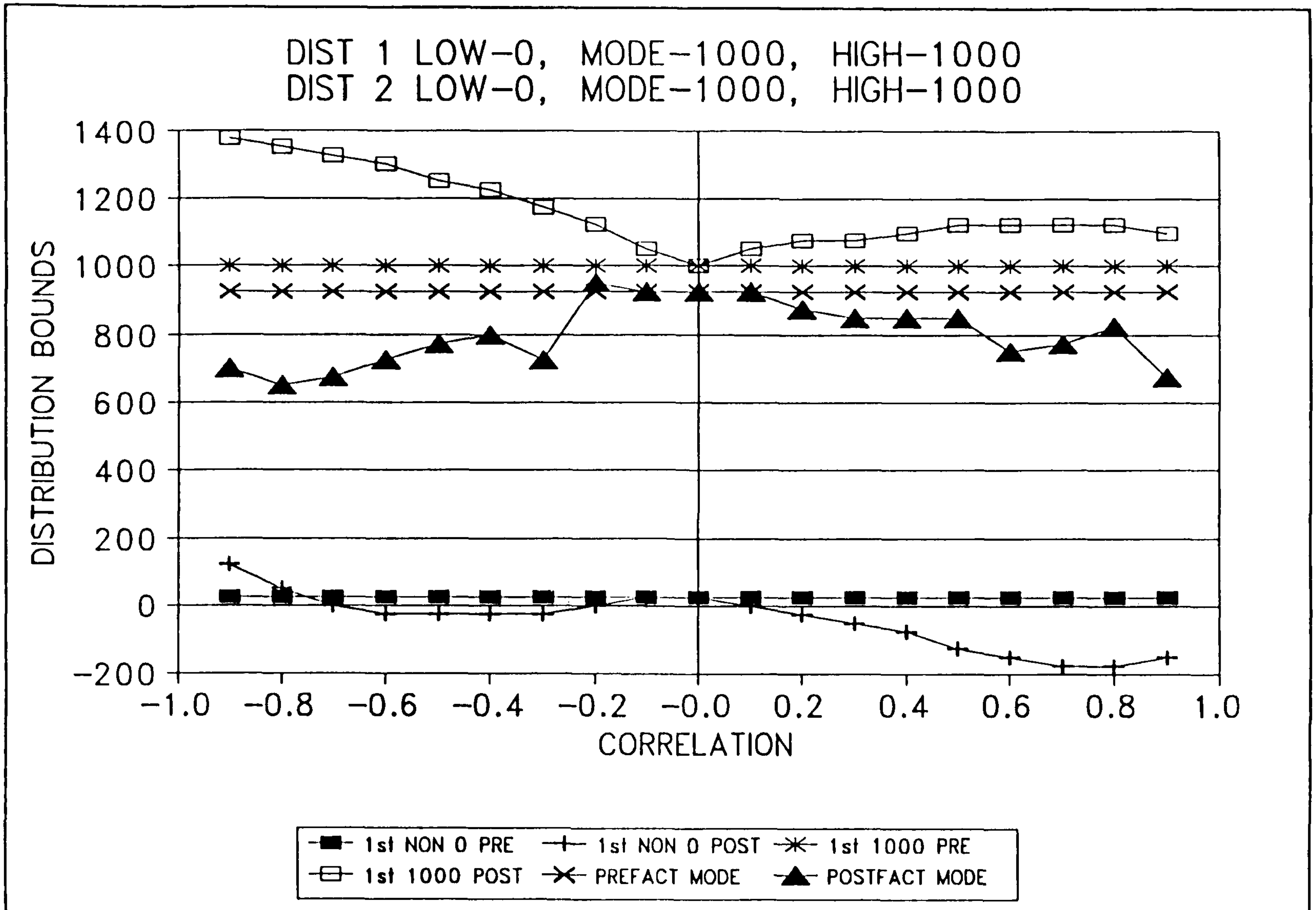


Figure 70 Boundary chart for Case 25 with random seed 1

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Thomas R. O'Hara, Capt, USAF

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This study develops a dependent component cost risk model validation methodology and applies it to the Air Force Risk Model. The validation process consists of ensuring that logically consistent input parameters are acted on properly by the cost risk methodology. Users of all dependent component risk models must be concerned with logically consistent input parameters. Two criteria define logical consistency. The first is the correlation matrix consistency and the second is the consistency between pairs of cost distributions. Three validation criteria are defined and used to validate a cost risk model. The first criterion is that the process must maintain the user defined correlations. The second criterion is that the total cost distribution mean and variance be congruous with the analytical value. The third criterion is that properly specified input parameters not be altered by the cost risk process. A Comparison Model was developed in Quattro Pro to validate the general Air Force Risk Model methodology. Twenty-five pairs of work breakdown structure cost elements are defined and tested in the Comparison Model. The final research product is a table illustrating the narrow conditions where the Air Force Risk Model is valid.

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